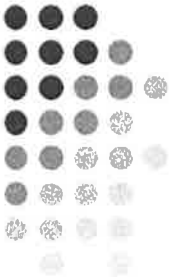
PreCalculus

Section 6.1

Combination of Vectors

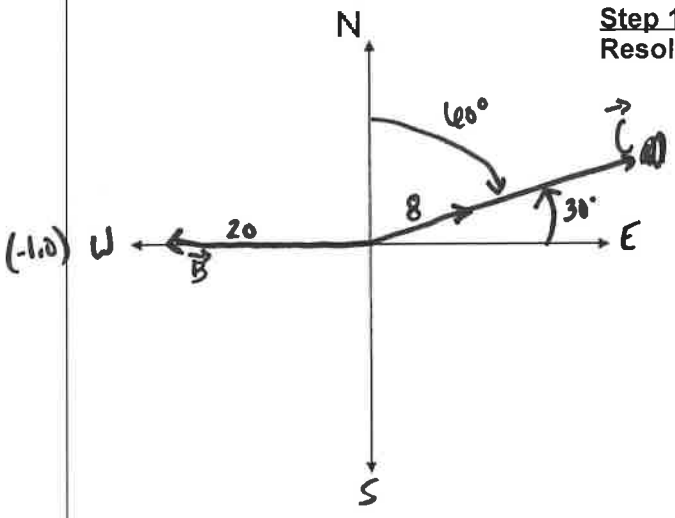
Resultant Vectors



HPC/RPC 2017

Apr 17-11:41 AM

Example 1: A boat is traveling due West at a speed of 20 mph. There is an 8 mph current flowing at a bearing of 60°. What is the "true" speed of the boat?



Step 1:
Resolve each vector into its components

$\vec{B} = \langle 20\cos 180, 20\sin 180 \rangle$

$\vec{B} = \langle -20, 0 \rangle$

$\vec{C} = \langle 8\cos 30, 8\sin 30 \rangle$

$\vec{C} = \langle 6.93, 4 \rangle$

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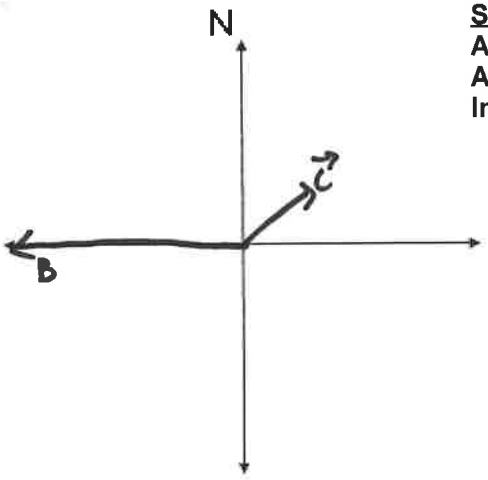
Example 1: A boat is traveling due West at a speed of 20 mph. There is an 8 mph current flowing at a bearing of 60°. What is the "true" speed of the boat?



Step 2:
Add horizontal components;
Add vertical components.
In other words; add the vectors

$\vec{B} + \vec{C} = \langle -20, 0 \rangle + \langle 6.93, 4 \rangle$

$\vec{R} = \langle -13.07, 4 \rangle$

$\|\vec{R}\| = \sqrt{(13.07)^2 + 4^2}$

$= 13.67 \text{ mph}$ Real speed

So, the boat's "true" speed due West is 13.07 without compensating for the current it's traveling at 4mph North.

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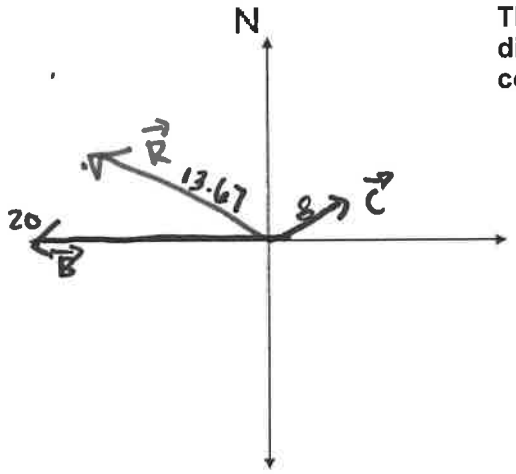
Example 1: A boat is traveling due West at a speed of 20 mph. There is an 8 mph current flowing at a bearing of 60°. What is the "true" speed of the boat?



The resultant vector shows the actual direction of the boat, without compensating for the current

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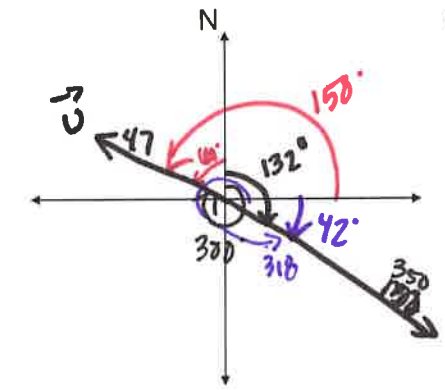
Example 2: An airplane is flying 350 mph at a bearing of 132°.
If the wind is blowing 47 mph at a bearing of 300°, find:

- the actual ("ground") speed of the plane and
- the bearing of the plane as a result of the wind



$\vec{P} = \langle 350\cos 318, 350\sin 318 \rangle$

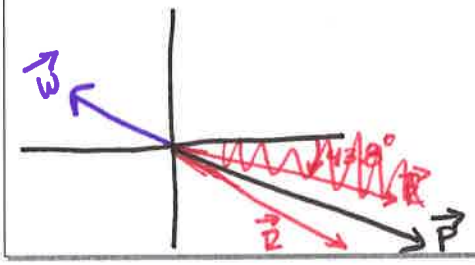
$= \langle 260.10, -234.20 \rangle$

$\vec{w} = \langle 47\cos 150, 47\sin 150 \rangle$

$= \langle -40.70, 23.5 \rangle$

$\vec{R} = \langle 219.4, -210.7 \rangle$

$\|\vec{R}\| = \sqrt{219.4^2 + (-210.7)^2} = 304.2$ Actual speed



$\theta = \cos^{-1}\left(\frac{219.4}{304.2}\right) = 43.8°$

Bearing $= 90 + 43.8 = \boxed{133.8°}$

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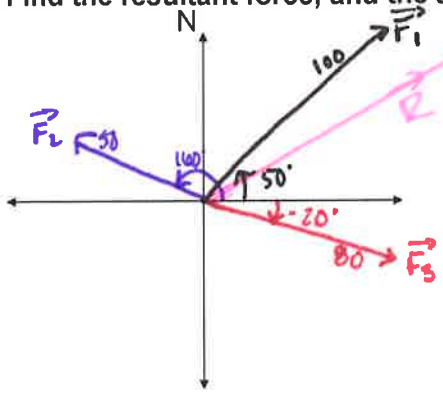
Example 3: Three forces all work on the same object.
Force 1 = 100 lb at an angle of 50°
Force 2 = 50 lb at an angle of 160°
Force 3 = 80 lb at an angle of -20° (or 340°)
Find the resultant force, and the angle at which it acts.



$F_1 = \langle 100\cos 50, 100\sin 50 \rangle$

$= \langle 64.28, 76.6 \rangle$

$F_2 = \langle 50\cos 160, 50\sin 160 \rangle$

$= \langle -46.98, 17.10 \rangle$

$F_3 = \langle 80\cos 340, 80\sin 340 \rangle$

$= \langle 75.18, -27.36 \rangle$

$\vec{R} = \langle 92.47, 66.34 \rangle$

$\|\vec{R}\| = \sqrt{92.47^2 + 66.34^2} = 113.8$

$\theta = \cos^{-1}\left(\frac{92.47}{113.8}\right) = 35.7°$

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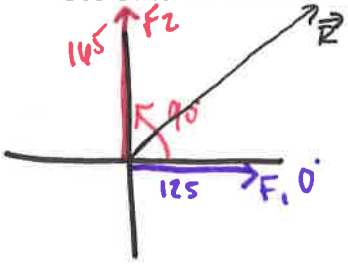
Vector Practice Problems

Name_____

Draw vector diagrams to solve each problem.

1) Two boys push on a box. One pushes with a force of 125 N to the east. The other exerts a force of 165 N to the north. What is the size and direction of the resultant force on the box?



$F_1 = \langle 125\cos 0, 125\sin 0 \rangle$

$= \langle 125, 0 \rangle$

$F_2 = \langle 165\cos 90, 165\sin 90 \rangle$

$= \langle 0, 165 \rangle$

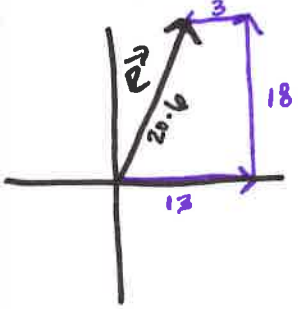
$\vec{R} = \langle 125, 165 \rangle$

$\|R\| = \sqrt{125^2 + 165^2}$

$= 207.002$

$\theta = \cos^{-1}\left(\frac{125}{207}\right) = 52.9°$

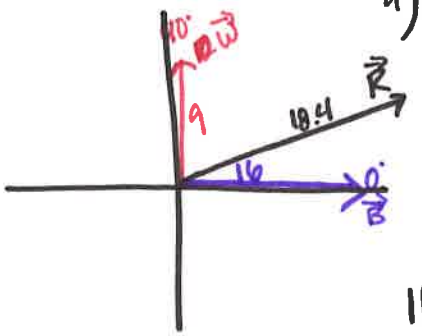
2) An explorer walks 13 km due east, then 18 km north, and finally 3 km west.
   a) What is the total distance walked? $13+18+3 = 34$ km
   b) What is the displacement of the explorer (current distance from the starting point)?



$\vec{R} = \langle 10, 18 \rangle$

$\|R\| = \sqrt{10^2 + 18^2} = 20.6$ km

3) A motorboat heads due east at 16 m/s across a river that flows due north at 9.0 m/s.
   a) What is the resultant velocity (speed and direction) of the boat?
   b) If the river is 136 m wide, how long does it take the motorboat to reach the other side?
   c) How far downstream is the boat when it reaches the other side of the river?



a) $\vec{B} = \langle 16, 0 \rangle$

$\vec{W} = \langle 0, 9 \rangle$

$\vec{R} = \langle 16, 9 \rangle$

$\|R\| = \sqrt{16^2 + 9^2} = 18.4$ m/s

$\theta = \cos^{-1}\left(\frac{16}{18.4}\right) = 29.4°$

b) $d = rt$

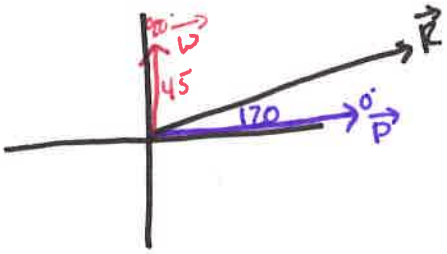
$\frac{136}{16} = \frac{16t}{16}$

$8.5\text{ s} = t$

c) $d = rt$

$d =$

4) While flying due east at 120 km/h, an airplane is also carried northward at 45 km/h by the wind blowing due north. What is the plane's resultant velocity?



$\vec{P} = \langle 120, 0 \rangle$

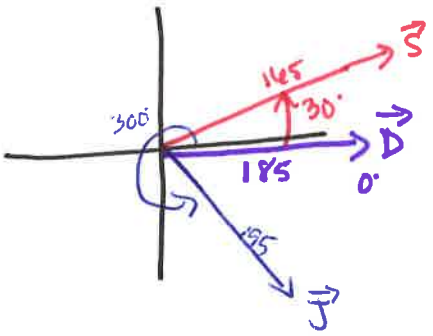
$\vec{W} = \langle 0, 45 \rangle$

$\vec{R} = \langle 120, 45 \rangle$

$\|R\| = \sqrt{120^2 + 45^2} = 128.16$ km/h Resultant velocity

$\theta = \cos^{-1}\left(\frac{120}{128.16}\right) = 20.56°$

5) Three teenagers push a heavy crate across the floor. Dion pushes with a force of 185 N at 0°. Shirley exerts a force of 165 N at 30°, while Joan pushes with 195 N force at 300°. What is the resultant force on the crate? What direction (angle) does the crate go?



$\vec{D} = \langle 185, 0 \rangle$

$\vec{S} = \langle 165\cos 30, 165\sin 30 \rangle$
$= \langle 142.89, 82.5 \rangle$

$\vec{J} = \langle 195\cos 300°, 195\sin 300 \rangle$
$= \langle 97.5, -168.88 \rangle$

$\vec{R} = \langle 425.4, -86.4 \rangle$

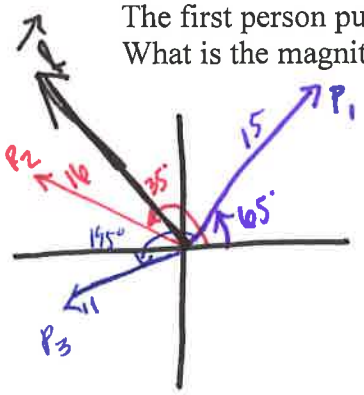
$\|R\| = \sqrt{425.4^2 + (-86.4)^2}$
$= 434.1$ N

$\theta = \cos^{-1}\left(\frac{425.4}{434.1}\right)$
$= 11.5°$

$\theta = 360 - 11.5$

$\theta \approx 348.5°$

6) Three people are pulling on a tree.
The first person pulls with 15 N at 65°; the second with 16 N at 135°; the third with 11 N at 195°.
What is the magnitude and direction of the resultant force on the tree?



$P_1 = \langle 15\cos 65, 15\sin 65 \rangle$
$= \langle 6.34, 13.6 \rangle$

$P_2 = \langle 16\cos 135, 16\sin 135 \rangle$
$= \langle -11.3, 11.3 \rangle$

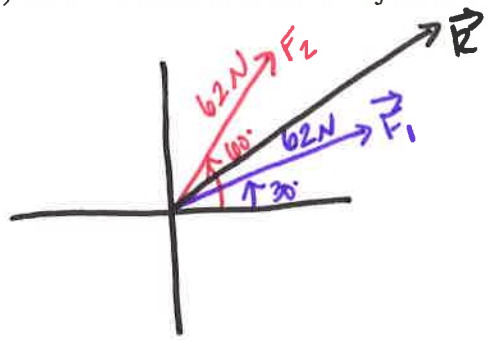
$P_3 = \langle 11\cos 195, 11\sin 195 \rangle$
$= \langle -10.6, -2.8 \rangle$

$\vec{R} = \langle -15.6, 22.1 \rangle$

$\|R\| = \sqrt{(-15.6)^2 + 22.1^2}$
$= 27.02$ N

$\theta = \cos^{-1}\left(\frac{-15.6}{27.02}\right) = 125.3°$

7) A 62-N force acts on an object at 30° and a second 62-N force acts at 60°. Determine the resultant force.



$F_1 = \langle 62\cos 30, 62\sin 30 \rangle$

$= \langle 53.7, 31 \rangle$

$F_2 = \langle 62\cos 60, 62\sin 60 \rangle$

$= \langle 31, 53.7 \rangle$

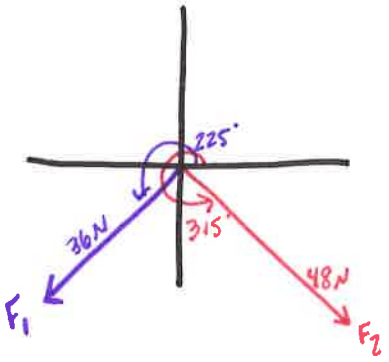
$\vec{R} = \langle 84.7, 84.7 \rangle$

$\|R\| = \sqrt{84.7^2 + 84.7^2} = 119.8\ N$

$\theta = \cos^{-1}\left(\frac{84.7}{119.8}\right)$

$= 45°$

8) Two forces act on an object. A 36-N force acts at 225°. A 48-N force acts at 315°. What would be the magnitude and direction of the resultant force?



$F_1 = \langle 36\cos 225, 36\sin 225 \rangle$

$= \langle -25.5, -25.5 \rangle$

$F_2 = \langle 48\cos 315, 48\sin 315 \rangle$

$= \langle 33.9, -33.9 \rangle$

$\vec{R} = \langle 8.5, -59.4 \rangle$

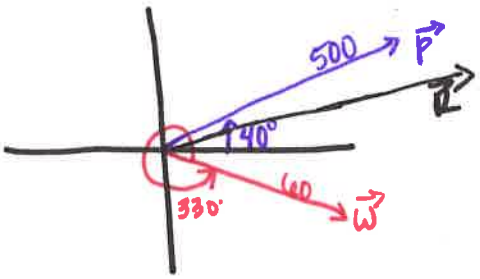
$\|R\| = \sqrt{8.5^2 + (-59.4)^2} = 60\ N$

$\theta = \cos^{-1}\left(\frac{8.5}{60}\right) = 81.9°$

$\theta = 360 - 81.9 = 278.13°$

Vector Word Problems Practice Worksheet 2 (Bearings)

1. An airplane has an airspeed of 500 kph bearing 50°. The wind velocity is 60 kph in the direction of 330°. Find the resultant vector representing the path of the plane relative to the ground. What is the ground speed of the plane? What is the direction?



$\vec{P} = \langle 500\cos 40, 500\sin 40 \rangle$

$= \langle 383.02, 321.39 \rangle$

$\vec{w} = \langle 60\cos 330, 60\sin 330 \rangle$

$= \langle 51.96, -30 \rangle$

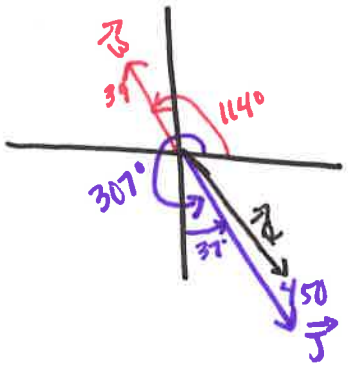
$\vec{R} = \langle 434.98, 291.39 \rangle$

$\|R\| = \sqrt{434.98^2 + 291.39^2} = 523.56$ kph

$\theta = \cos^{-1}\left(\frac{434.98}{523.56}\right)$

$= 33.8°$

2. An airline route from San Francisco to Honolulu is S 37° E. A jet flying at 450 mph on that bearing runs into a wind blowing at 39 mph from a direction of 114 degrees. Find the resulting groundspeed and direction of the jet.



$\vec{J} = \langle 450\cos 307, 450\sin 307 \rangle$

$= \langle 270.82, -359.39 \rangle$

$\vec{w} = \langle 39\cos 114, 39\sin 114 \rangle$

$= \langle -15.9, 35.6 \rangle$

$\vec{R} = \langle 254.95, -323.76 \rangle$

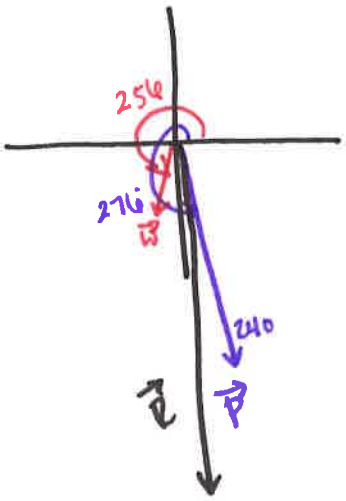
$\|R\| = \sqrt{254.95^2 + (-323.76)^2} = 412.09$ mph

$\theta = \cos^{-1}\left(\frac{254.95}{412.09}\right)$

$= 51.8°$

$\theta = 360 - 51.8 = 308.2°$

3. An airplane is heading S 6° E at an airspeed of 240 kph. A 20 kph wind is blowing from S 14° W. Find the groundspeed and resultant direction of the plane.



$\vec{P} = \langle 240\cos 276, 240\sin 276 \rangle$

$= \langle 25.09, -238.69 \rangle$

$\vec{w} = \langle 20\cos 256, 20\sin 256 \rangle$

$= \langle -4.84, -19.41 \rangle$

$\vec{R} = \langle 20.2, -258.1 \rangle$

$\|R\| = \sqrt{20.2^2 + (-258.1)^2} = 258.9$ kph

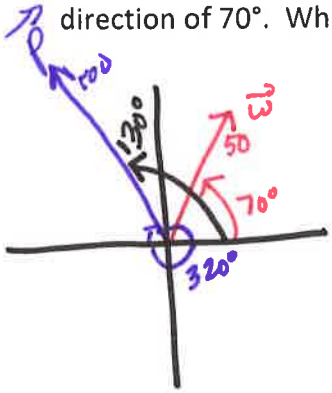
$\theta = \cos^{-1}\left(\frac{20.2}{258.9}\right)$

$= 85.5°$

$\theta = 360 - 85.5$

$= 274.5°$

4. An airplane is traveling at a speed of 500 mph with a bearing of 320° at a fixed altitude and no wind. As the plane crosses the Mississippi river, it encounters a wind blowing with a velocity of 50 mph in the direction of 70°. What is the resultant speed and direction of the plane?



$\vec{p} = \langle 500\cos 130, 500\sin 130 \rangle$

$= \langle -321.39, 383.02 \rangle$

$\vec{w} = \langle 50\cos 70, 50\sin 70 \rangle$

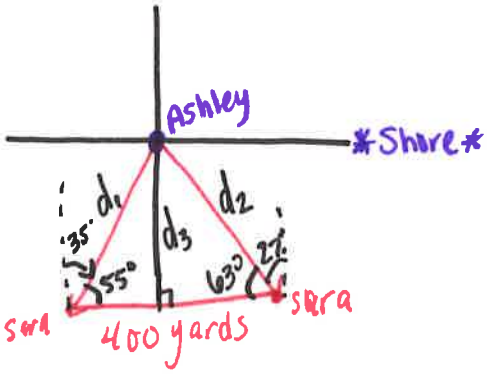
$= \langle 17.10, 46.98 \rangle$

$\vec{R} = \langle -304.29, 430.01 \rangle$

$\|R\| = \sqrt{(-304.29)^2 + 430.01^2} = \boxed{526.8 \text{ mph}}$

$\theta = \cos^{-1}\left(\frac{-304.29}{526.8}\right) = \boxed{125.3°}$

5. Sara is in a boat traveling due west parallel to the shore. At one point Sara sees her friend Ashley on the shore at a bearing of S 35° W. Sara continues west for 400 more yards, where now she sees her friend at a bearing of S 27° E. How far is Sara from Ashley at both points? How far is Sara from the shore?



Law of Sines

$\frac{d_1}{\sin 63°} = \frac{400}{\sin 62°}$

$d_1 = 403.65 \text{ yds.}$

$\frac{d_2}{\sin 55°} = \frac{400}{\sin 62°}$

$d_2 = 371.1 \text{ yd.}$

$\sin 63° = \frac{d_3}{371.1}$

$d_3 = 330.65 \text{ yds.}$