Pre-Calculus Vector Review

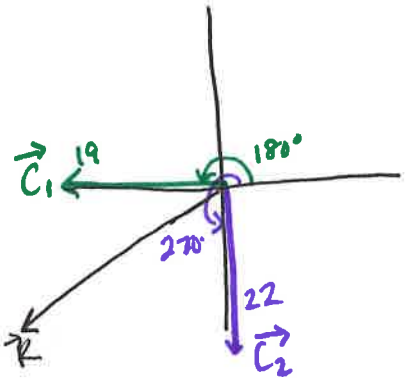
Name KEY

Coordinate Systems

Level 1

Solve the problems below. Sketch all vectors, and use proper notation in all answers.

1) One child pulls a wagon directly west with a force of 19 newtons, and another child pulls the wagon directly south with a force of 22 newtons. Find the resultant vector of the wagon.

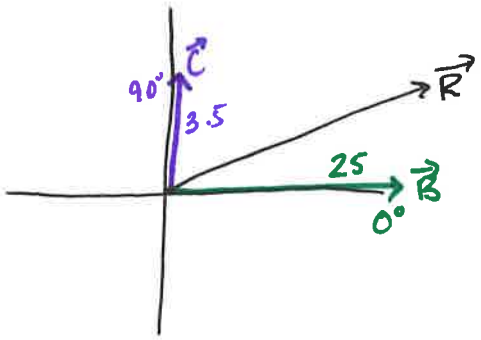


$\vec{C_1} = \langle 19\cos 180^\circ, 19\sin 180^\circ \rangle = \langle -19, 0 \rangle$

$\vec{C_2} = \langle 22\cos 270^\circ, 22\sin 270^\circ \rangle = \langle 0, -22 \rangle$

$\vec{C_1 + C_2} = \langle -19, -22 \rangle$

2) A boat is travelling due east at a speed of 25 mph. The current is flowing due north at a speed of 3.5 mph. Find the actual magnitude of the boat.



$\vec{B} = \langle 25\cos 0^\circ, 25\sin 0^\circ \rangle = \langle 25, 0 \rangle$

$\vec{C} = \langle 3.5\cos 90^\circ, 3.5\sin 90^\circ \rangle = \langle 0, 3.5 \rangle$

$\vec{B + C} = \langle 25, 3.5 \rangle$

3) A tow truck is pulling a car with a force of 117 pounds. How much work is done in moving the car 50 feet if the angle of the road 12° with the horizontal?

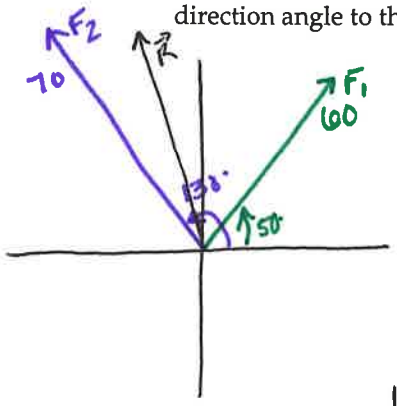
$W = 117(50)\cos 12^\circ$

$W = 5722.16$ ft lbs.

(Levels 2/3)

Solve the problems below. Sketch all vectors, and use proper notation in all answers.

4) Two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$, of magnitude 60 and 70 pounds, respectively, act on an object. The direction of $\mathbf{F_1}$ is 50° and the direction of $\mathbf{F_2}$ is 130°. Find the magnitude and the direction angle of the resultant force. Express the direction angle to the nearest tenth of a degree.



$F_1 = \langle 60\cos 50°, 60\sin 50° \rangle = \langle 38.567, 45.963 \rangle$

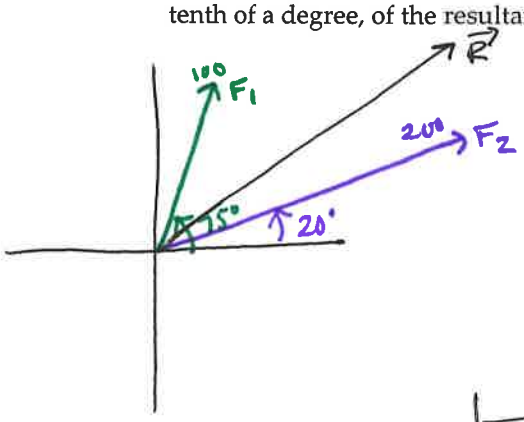
$F_2 = \langle 70\cos 130°, 70\sin 130° \rangle = \langle -44.995, 53.623 \rangle$

$\overrightarrow{F_1+F_2} = \langle -6.428, 99.586 \rangle$

$|F_1+F_2| = \sqrt{(-6.428)^2 + 99.586^2} = 99.8 \text{ lbs}$

$\theta = \cos^{-1}\left(\frac{-6.428}{99.8}\right) \approx 93.7°$

5) The magnitude and direction of two forces acting on an object are 100 pounds, 75° and 200 pounds, 20°, respectively. Find the magnitude, to the nearest hundredth of a pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.



$F_1 = \langle 100\cos 75, 100\sin 75 \rangle = \langle 25.882, 96.593 \rangle$

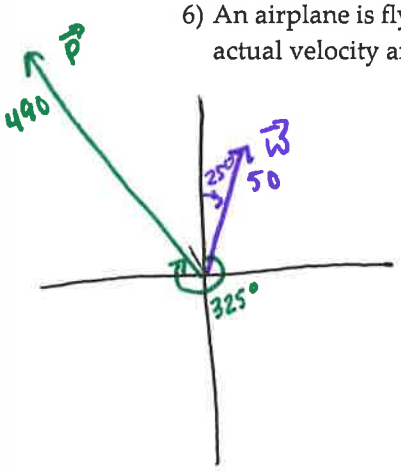
$F_2 = \langle 200\cos 20, 200\sin 20 \rangle = \langle 187.939, 68.404 \rangle$

$\overrightarrow{F_1+F_2} = \langle 213.82, 164.997 \rangle$

$|\overrightarrow{F_1+F_2}| = \sqrt{213.82^2 + 164.997^2} = 270.1 \text{ lbs}$

$\theta = \cos^{-1}\left(\frac{213.82}{270.1}\right) \approx 37.7°$

6) An airplane is flying on a bearing of 325° at 490 mph. It encounters a 50 mph wind at a bearing of 25°. Find the actual velocity and direction of the airplane.



$\vec{p} = \langle 490\cos 125°, 490\sin 125° \rangle = \langle -281.052, 401.385 \rangle$

$\vec{w} = \langle 50\cos 65°, 50\sin 65° \rangle = \langle 21.131, 45.315 \rangle$

$\vec{p}+\vec{w} = \langle -259.922, 446.7 \rangle$

$|\vec{p}+\vec{w}| = \sqrt{(-259.922)^2 + 446.7^2} \approx 516.8 \text{ mph}$

$\theta = \cos^{-1}\left(\frac{-259.922}{516.8}\right) \approx 120.2°$

7) Find the angle between the given vectors to the nearest tenth of a degree and determine whether they are parallel, orthogonal or neither.

a) $\mathbf{u} = \langle 6, 3 \rangle$, $\mathbf{v} = \langle -5, -3 \rangle$

$u \cdot v = 6(-5) + 3(-3) = -39$

$\|u\| = \sqrt{6^2 + (3)^2} = \sqrt{45}$

$\|v\| = \sqrt{(-5)^2 + (-3)^2} = \sqrt{34}$

$\theta = \cos^{-1}\left(\frac{-39}{\sqrt{45}\cdot\sqrt{34}}\right) \approx 175.6°$ neither

b) $\mathbf{u} = \langle 4, -2 \rangle$, $\mathbf{v} = \langle 6, 12 \rangle$

$u \cdot v = 4(6) + (-2)(12) = 0$

$\|u\|$

$\|v\|$

$\theta = \cos^{-1}(0) = 90°$

Orthogonal

c) $\mathbf{u} = \langle 4, -6 \rangle$, $\mathbf{v} = \langle -8, 12 \rangle$

$u \cdot v = 4(-8) + (-6)(12) = -104$

$\|u\| = \sqrt{4^2 + (-6)^2} = \sqrt{52}$

$\|v\| = \sqrt{(-8)^2 + 12^2} = \sqrt{208}$

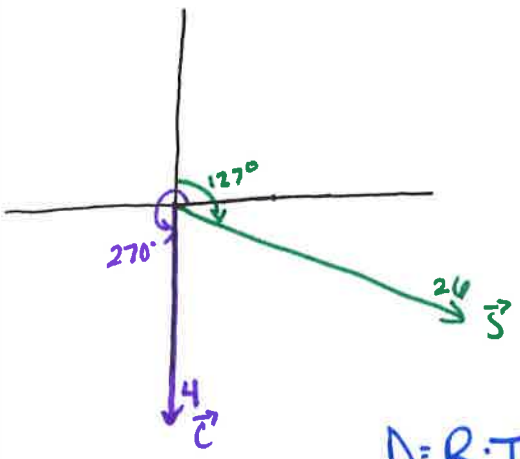
$\theta = \cos^{-1}\left(\frac{-104}{\sqrt{52}\cdot\sqrt{208}}\right) = 180°$ parallel

8) Find the work done lifting a 300 pound boulder 55 feet into the air.

$\text{Work} = 300\cos 0°\,(55)$

$W = 16500 \text{ ft. lbs.}$

9) A cruise ship going from Miami to Nassau on a bearing of 127° is travelling at a speed of 26 miles per hour. The current is moving south at a speed of 4 miles per hour. Find the actual speed and direction of the ship. If the total distance between Miami and Nassau is about 187 miles, how long will the cruise take?



$\vec{S} = \langle 26\cos 323°, 26\sin 323° \rangle = \langle 20.765, -15.647 \rangle$

$\vec{C} = \langle 4\cos 270°, 4\sin 270° \rangle = \langle 0, -4 \rangle$

$\vec{S+C} = \langle 20.765, -19.647 \rangle$

$\|\vec{S+C}\| = \sqrt{20.765^2 + (-19.647)^2} = 28.6 \text{ mph}$

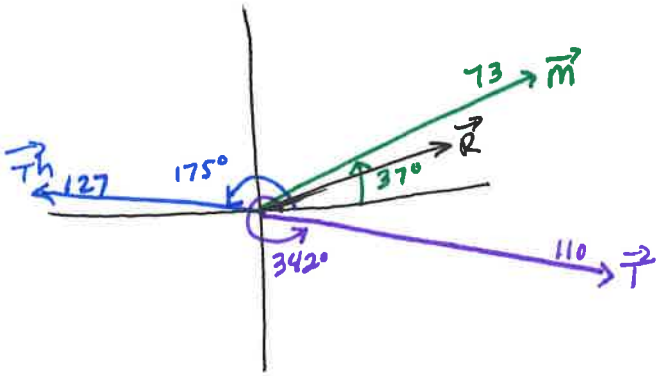
$\theta = \cos^{-1}\left(\frac{20.765}{28.6}\right) \approx 43.4° \rightarrow 360 - 43.4 = 316.6°$

$D = R \cdot T$

$\frac{187}{28.6} = \frac{28.6T}{28.6}$

$6.5 \text{ hrs} = T$

10) Maria and Tina are walking their dog, Thor. Thor does not like to be on the leash and is pulling the girls the opposite direction. Maria is pulling with a force of 73 lb at a 37° angle, Tina is pulling with a force of 110 lb at 342°, and Thor is using a force of 127 lb at 175°. Who is really being taken for a walk? How do you know?



$\vec{M} = \langle 73\cos 37°, 73\sin 37° \rangle = \langle 58.300, 43.932 \rangle$

$\vec{T} = \langle 110\cos 342°, 110\sin 342° \rangle = \langle 104.616, -33.992 \rangle$

$\vec{Th} = \langle 127\cos 175°, 127\sin 175° \rangle = \langle -126.517, 11.069 \rangle$

$\vec{R} = \langle 36.399, 21.009 \rangle$

$\|\vec{R}\| = \sqrt{36.399^2 + 21.009^2} \approx 42.027 \text{ lbs}$

$\theta = \cos^{-1}\left(\frac{36.399}{42.027}\right) \approx 30°$

The girls are walking the dog... but it is pretty slow because their resultant ~~force~~ magnitude is only 42 lbs.