**Exponential/Logarithmic Functions and Their Graphs** **Name** Key

**Pre-Calculus Notes** **Period** _____

The standard form of an exponential function is $f(x) =$ $a \cdot b^x$ (2, 3...e any #) where $a \neq 0$ and $b > 0; b \neq 1$.

Logarithmic functions have a standard form $f(x) =$ $\log_b x$ or $\ln x$ where $x > 0$ and $b > 0; b \neq 1$.

Exponential and logarithmic functions are inverses of one another.

| | Equation | Description |
|---|---|---|
| **Vertical Translation (shift up/down)** | $y = f(x) + c$ OR $y = f(x) - c$ | The function, *f(x)*, is shifted up (+) or down (-) by "c" units. |
| **Horizontal Translation (shift left/right)** | $y = f(x + c)$ OR $y = f(x - c)$ | The function, *f(x)*, is shifted left (+) or right (-) by "c" units. |
| **Reflection** | $y = -f(x)$ OR $y = f(-x)$ | The function, *f(x)*, is reflected over the x- axis (*-f(x)*) or the y- axis (*f(-x)*). |
| **Vertical Stretch or Shrink** | $y = cf(x)$ | If c >1; *f(x)* stretched vertically<br>If 0<c <1; *f(x)* shrunk vertically |
| **Horizontal Stretch or Shrink** | $y = f(cx)$ | If c >1; *f(x)* shrunk horizontally<br>If 0<c <1; *f(x)* stretched horizontally |

Example 1: Given $f(x) = e^x$, describe all of the transformations that will occur between *f(x)* and *g(x)*.

a) $g(x) = -e^x$

Reflect over X-axis

b) $g(x) = e^{-2x}$

Reflect over y-axis

Horizontal shrink by factor of 2

c) $g(x) = \frac{1}{4}e^{x+3}$

Vertical shrink by factor of 4

Left 3

d) $g(x) = -3e^{x-1} + 5$

Reflect over X-axis

Vertical stretch by factor of 3

Right 1

up 5

Example 2: Given $f(x) = \log x$, describe all of the transformations that will occur between *f(x)* and *g(x)*.

a) $g(x) = \log 5x$

Horizontal shrink by factor of 5

b) $g(x) = -\frac{1}{2}\log x$

Reflect over X-axis

Vertical shrink by factor of 2

c) $g(x) = \log(-x) - 4$

Reflect over y-axis

Down 4

d) $g(x) = \log(-5x + 2)$

Reflect over y-axis

Horizontal shrink by factor of 5

Left 2

Example 3: Given $f(x) = 2^x$, write the equation for the transformed function, *g(x)*.

a) Transform *f(x)* by shifting up 10 units and reflecting over the x-axis.

$g(x) = -2^x + 10$

b) Transform *f(x)* by shifting left 8 units, horizontally stretching by a factor of 3 and reflecting over the y-axis.

$g(x) = 2^{-\frac{1}{3}x + 8}$

**Pre-Calculus: Exponential/Log functions** Name _____

**Day 1** Date_____ Per_____

Given the following *f(x)* and *g(x)* equations, determine all transformations that occur between the two.

1. $f(x) = 3^x$

a) $g(x) = -(3^x)$

Reflect over x-axis

b) $g(x) = 3^{2x}$

Horizontal shrink by 2

2. $f(x) = \log_2 x$

a) $g(x) = \log_2(-x - 4)$

Reflect over y-axis

Right 4

b) $g(x) = \frac{1}{2}\log_2 x + 6$ ← No parenthesis so up

Vertical shrink by 2

up 6

3. $f(x) = e^x$

a) $g(x) = e^{-0.5x} - 1$ ← 1/2

Reflect over y-axis

Horizontal stretch by 2

Down 1

b) $g(x) = -3e^{x+8}$

Reflect over x-axis

Vertical stretch by 3

Left 8

4. $f(x) = \ln x$

a) $-\ln(\frac{1}{5}x + 2) - 7$

Reflect over x-axis

Horizontal stretch by 5

Left 2

Down 7

b) $-4\ln(-x + 2)$

Reflect y-axis

Vertical stretch by 4

Reflect x-axis

Left 2

5. $f(x) = 4^x$

a) $g(x) = -2\,(4^{x-1})$

Reflect over X-axis

Vertical Stretch by 2

Right 1

b) $g(x) = 4^{-0.25x} + 5$ ← 1/4

Reflect over y-axis

Horizontal stretch by 4

up 5

6. Write an equation for the function that is described by the given characteristics.

a) The shape of $f(x) = e^x$, but moved two units to the right and eight units down.

$g(x) = e^{x-2} - 8$

b) The shape of $f(x) = 4^x$, but reflected over the x-axis and vertically stretched by a factor of 6.

$g(x) = -6(4^x)$

c) The shape of $f(x) = \log x$, but moved 5 units up and reflected about the y-axis.

$g(x) = \log(-x) + 5$

d) The shape of $f(x) = \log_3 x$, but horizontally shrunk by a factor of 2 and shifted up 3 units.

$g(x) = \log_3(2x) + 3$

e) The shape of $f(x) = 3^x$, but stretched vertically by a factor of 2, stretched horizontally by a factor of 2, shifted down 2 units, and left 2 units.

$g(x) = 2\left(3^{\frac{1}{2}x+2}\right) - 2$