Cornell Notes	Topic/Objective:	Name:
1.5 Inverse Functions	*Students will determine if a function has an*	Class/Period:
	inverse & then find the inverse.	Date:

Essential Question:

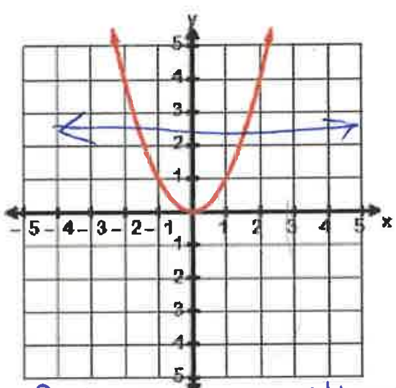
Questions:	Notes:

Horizontal Line Test (HLT):

If a horizontal line passes through the graph of a function two or more times (above $y=0$), then the function has no inverse

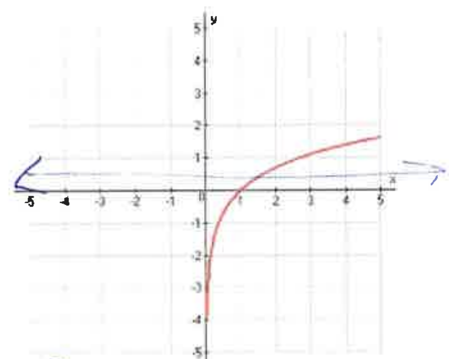
Example 1: Determine if the following functions have an inverse using the HLT.

a.



fails HLT, NO INVERSE

b.



Passes HLT, INVERSE

If a function passes the HLT, then inverse can be algebraically by following these steps:

1. Make sure equation is in $y=mx+b$ form, replace $f(x)$ with y
2. Switch x and y
3. Solve for y
4. Replace y with $f^{-1}(x)$ → inverse

Summary:

Questions:	Notes:

Example 1: Find an equation for $f^{-1}(x)$ if $f(x)=\dfrac{2x}{2x-1}$

① $y=\dfrac{2x}{2x-1}$

④ $f^{-1}(x)=\dfrac{x}{2x-2}$

② $x=\dfrac{2y}{2y-1}$

③ $x(2y-1)=2y$

$2xy-x=2y$

$2xy-2y=x$

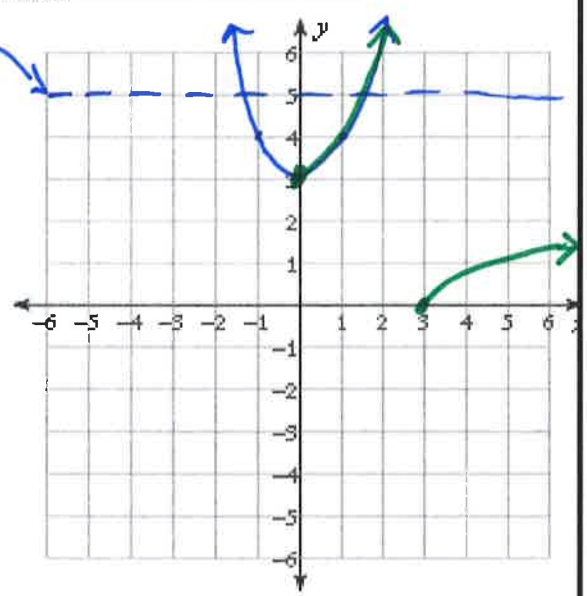
$y(2x-2)=x \longrightarrow y=\dfrac{x}{2x-2}$

If a function fails the HLT, then the domain needs to be restricted to produce an inverse.

Example 2: Graph $f(x)=x^2+3$. Determine if it is invertible. If it is, find the inverse. If it is not, restrict the domain and then find the inverse.

Not invertible, fails HLT

Domain Restriction: $[0,\infty)$



① $y=x^2+3$

② $x=y^2+3$

③ $x-3=y^2$

$\sqrt{x-3}=y$

④ $f^{-1}(x)=\sqrt{x-3}$

Summary:

Questions:	DAY 2 Notes:
How can I tell if functions are inverses of each other?	Two Methods: $f(f^{-1}(x)) = x$ or 4 steps

Example 3: Show that each function is the inverse of the other.

$f(x) = -2x + 1$

$g(x) = \frac{-x+1}{2}$

$f(g(x)) = -2\left(\frac{-x+1}{2}\right) + 1$

$= x - 1 + 1$

$= x$ ✓

OR

$y = -2x + 1$

$x = -2y + 1$

$x - 1 = -2y$

$\frac{x-1}{-2} = y$

$f^{-1}(x) = \frac{-x+1}{2}$ this matches $g(x)$, so they are inverses.

Summary:

Questions:	Notes:

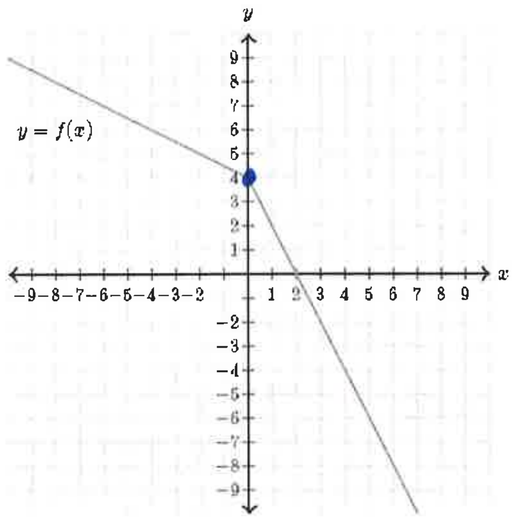
Example 4: Evaluate the following using the information given.

a) Given the table, find $f^{-1}(3)$. $= 9$

x	f(x)
4	-1
5	4
9	3

y instead of x

b) Given the graph, find $f^{-1}(4)$. $= 0$



c) Given the table, find $f^{-1}(8)$ AND $f^{-1}(f^{-1}(13))$.

x	-7	11	-13	6	5	-9
$f(x)$	7	12	8	-7	13	5

$f^{-1}(8) = -13$

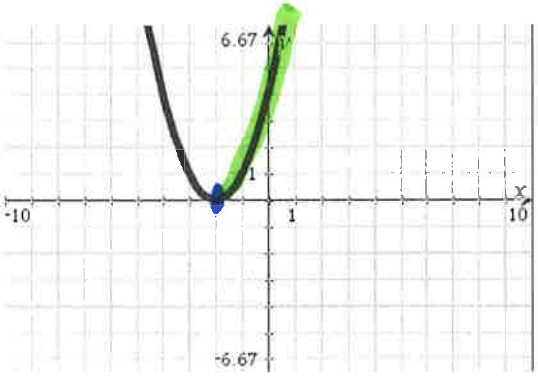
$f^{-1}(\underbrace{f^{-1}(13)}_{5}) = -9$

Summary:

HPC/RPC Inverse Worksheet: Name KEY

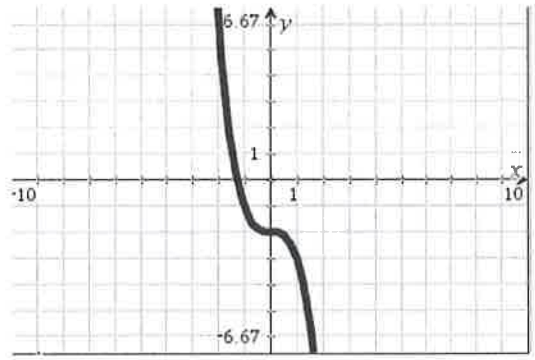
Day 1 Date ________ Per ____

For exercises 1-6, determine whether the function graphed is invertible. If it is not, restrict the domain that will make it invertible.

1. 

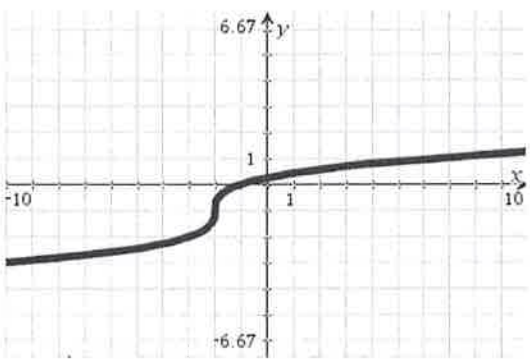
Invertible? NO

Restricted Domain? [-2, ∞)

2. 

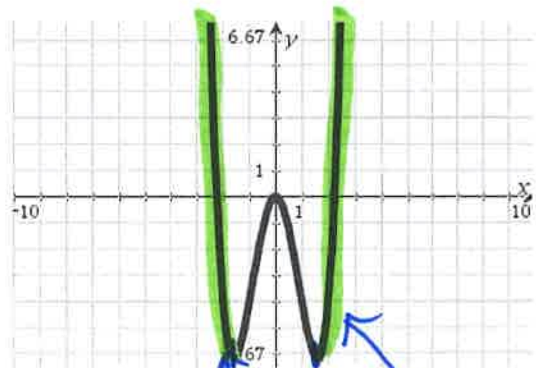
Invertible? yes

Restricted Domain? ________

3. 

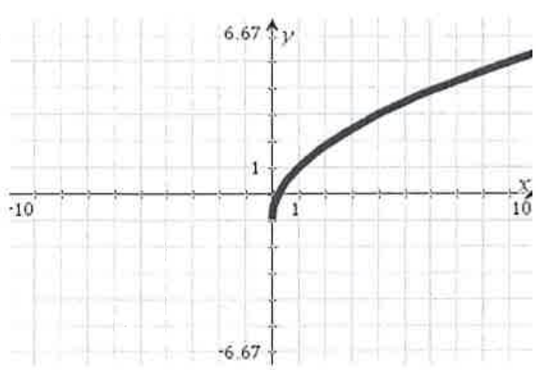
Invertible? yes

Restricted Domain? ________

4. 

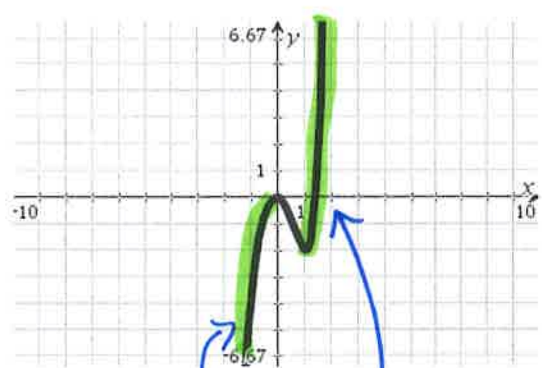
Invertible? NO

Restricted Domain? (-∞, -1.5] or [1.5, ∞)

5. 

Invertible? yes

Restricted Domain? ________

6. 

Invertible? NO

Restricted Domain? [1, ∞) OR (-∞, 0]

For Exercises 7-12, find an equation for $f^{-1}(x)$.

7. $f(x)=\dfrac{x}{x+1}$

$f^{-1}(x)=\dfrac{-x}{x-1}$ OR $\dfrac{x}{1-x}$

$y=\dfrac{x}{x+1}$

$x=\dfrac{y}{y+1}$

$x(y+1)=y$

$xy+x=y$

$xy-y=-x$

$y(x-1)=-x$

$\rightarrow y=\dfrac{-x}{x-1}$

8. $f(x)=\dfrac{x}{x-1}$

$f^{-1}(x)=\dfrac{x}{x-1}$

$y=\dfrac{x}{x-1}$

$x=\dfrac{y}{y-1}$

$xy-x=y$

$xy-y=x$

$y(x-1)=x$

$y=\dfrac{x}{x-1}$

9. $f(x)=\dfrac{2x}{3x-4}$

$f^{-1}(x)=\dfrac{4x}{3x-2}$

$y=\dfrac{2x}{3x-4}$

$x=\dfrac{2y}{3y-4}$

$x(3y-4)=2y$

$3xy-4x=2y$

$3xy-2y=4x$

$y(3x-2)=4x \rightarrow y=\dfrac{4x}{3x-2}$

10. $f(x)=\dfrac{x+1}{x-1}$

$f^{-1}(x)=\dfrac{x+1}{x-1}$

$y=\dfrac{x+1}{x-1}$

$x=\dfrac{y+1}{y-1}$

$x(y-1)=y+1$

$xy-x=y+1$

$xy-y=x+1$

$y(x-1)=x+1 \longrightarrow y=\dfrac{x+1}{x-1}$

11. $f(x)=\dfrac{2x+3}{x-3}$

$f^{-1}(x)=\dfrac{3x+3}{x-2}$

$y=\dfrac{2x+3}{x-3}$

$x=\dfrac{2y+3}{y-3}$

$xy-3x=2y+3$

$xy-2y=3x+3$

$y(x-2)=3x+3$

$y=\dfrac{3x+3}{x-2}$

12. $f(x)=\sqrt[3]{x-2}$

$f^{-1}(x)=x^3+2$

$y=\sqrt[3]{x-2}$

$x=\sqrt[3]{y-2}$

$x^3=y-2$

$x^3+2=y$

Precalculus – Inverses Day 2 Name:_______________ Date:_______

For each function:

a) Find the inverse of each function.
b) Graph the function and its inverse.
c) Use interval notation and find the domain and range of the function and its inverse.

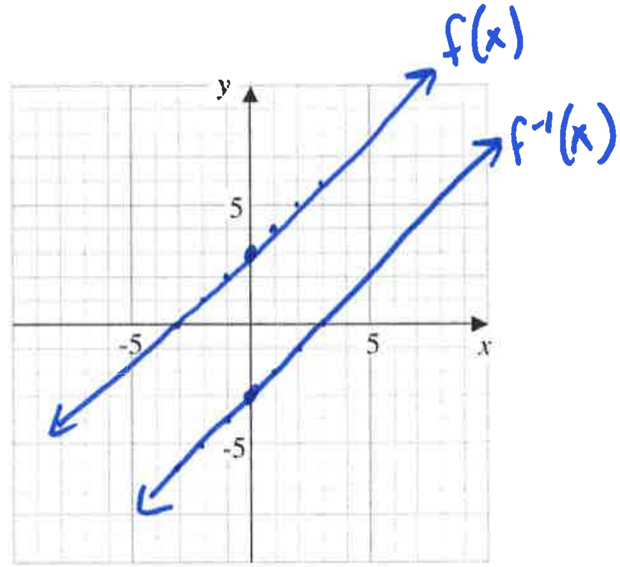
1. $f(x) = x + 3$

$y = x + 3$

$x = y + 3$

$x - 3 = y$

$f^{-1}(x) = x - 3$



Domain of $f(x) =$ $\mathbb{R}$

Domain of $f^{-1}(x) =$ $\mathbb{R}$

2. $f(x) = x^3 + 2$

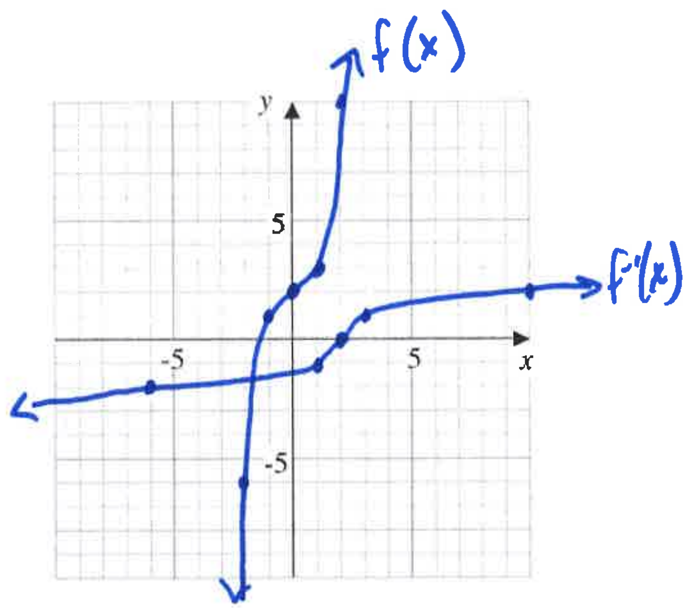
$y = x^3 + 2$

$x = y^3 + 2$

$x - 2 = y^3$

$\sqrt[3]{x-2} = y$

$f^{-1}(x) = \sqrt[3]{x-2}$

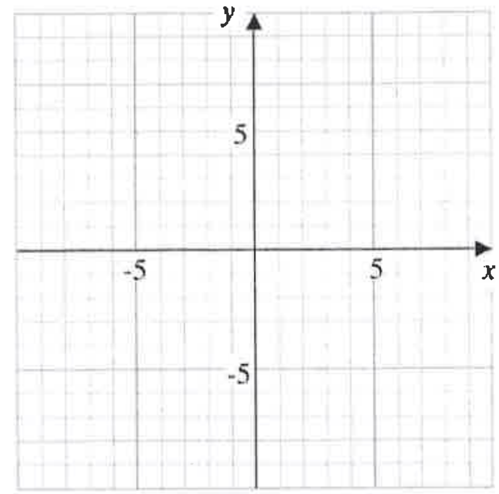


Domain of $f(x) =$ $\mathbb{R}$

Domain of $f^{-1}(x) =$ $\mathbb{R}$

3. $f(x) = x + 3$

Same as #1



Domain of $f(x) =$ __________

Domain of $f^{-1}(x) =$ __________

4. $f(x) = \sqrt[3]{x-1}$

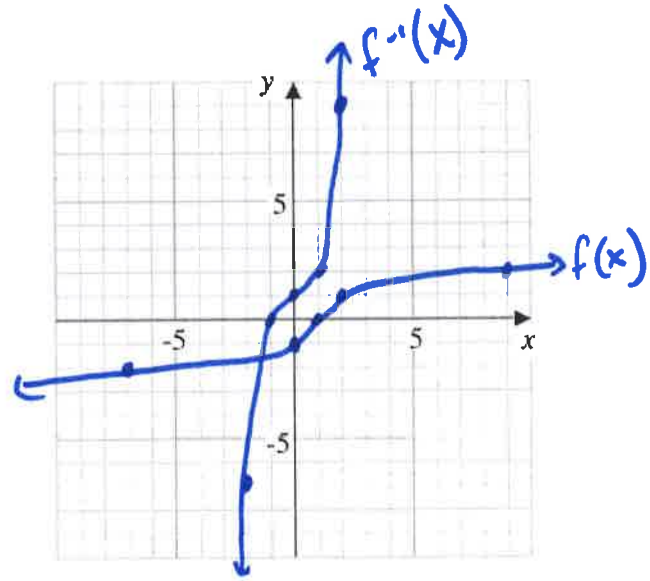
$y = \sqrt[3]{x-1}$

$x = \sqrt[3]{y-1}$

$x^3 = y - 1$

$x^3 + 1 = y$

$f^{-1}(x) = x^3 + 1$



Domain of $f(x) =$ $\mathbb{R}$

Domain of $f^{-1}(x) =$ $\mathbb{R}$

5. $f(x) = \frac{2x+1}{x-3}$ $\rightarrow$ $x-3 \neq 0$

$x \neq 3$

$y = \frac{2x+1}{x-3}$

$x = \frac{2y+1}{y-3}$

$x(y-3) = 2y+1$

$xy - 3x = 2y+1$

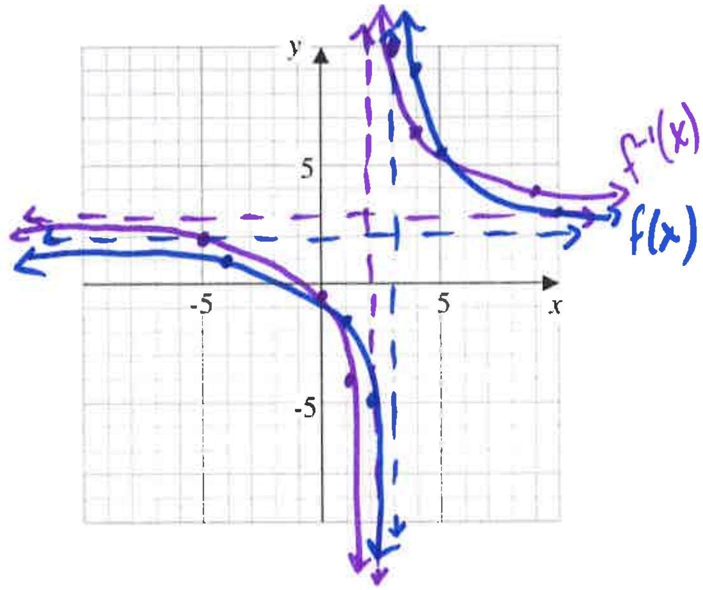
$xy - 2y = 3x+1$

$y(x-2) = 3x+1$

$y = \frac{3x+1}{x-2}$ $\quad f^{-1}(x) = \frac{3x+1}{x-2}$

$x-2 \neq 0$

$x \neq 2$



Domain of $f(x) =$ $(-\infty,3)\cup(3,\infty)$

Domain of $f^{-1}(x) =$ $(-\infty,2)\cup(2,\infty)$

6. $f(x) = \frac{2x-3}{x+1}$

$x+1 \neq 0$

$x \neq -1$

$y = \frac{2x-3}{x+1}$

$x = \frac{2y-3}{y+1}$

$x(y+1) = 2y-3$

$xy + x = 2y - 3$

$xy - 2x = -x - 3$

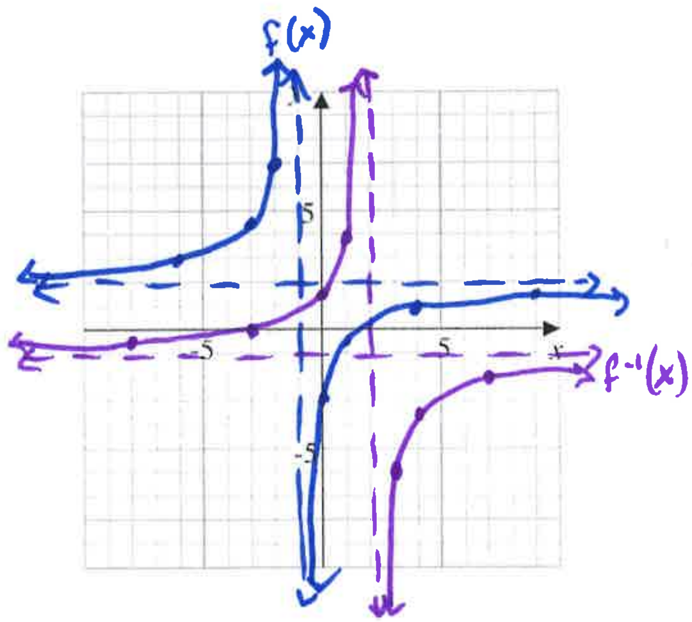
$y(x-2) = -1(x+3)$

$y = \frac{-(x+3)}{x-2}$

$f^{-1}(x) = \frac{-(x+3)}{x-2}$

$x-2 \neq 0$

$x \neq 2$



Domain of $f(x) =$ $(-\infty,-1)\cup(-1,\infty)$

Domain of $f^{-1}(x) =$ $(-\infty,2)\cup(2,\infty)$

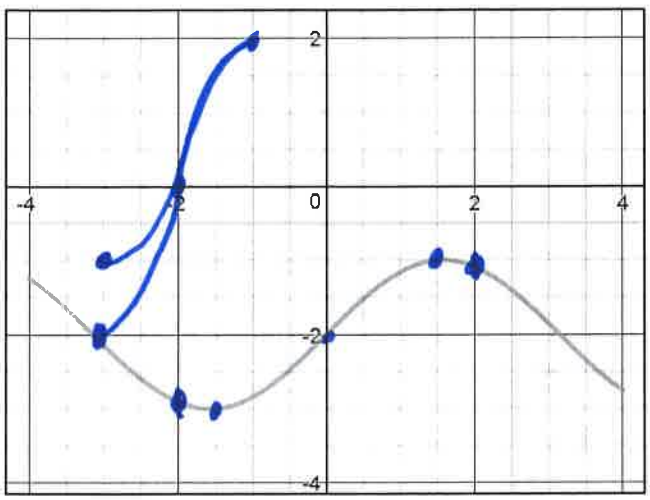
HPC/RPC Inverses Day 3:

Name KEY

Date ______________ Per ______

For exercises 1-4, determine whether the function graphed is invertible. If it is not, restrict the domain to make it invertible, and graph the inverse based on that domain.

1.

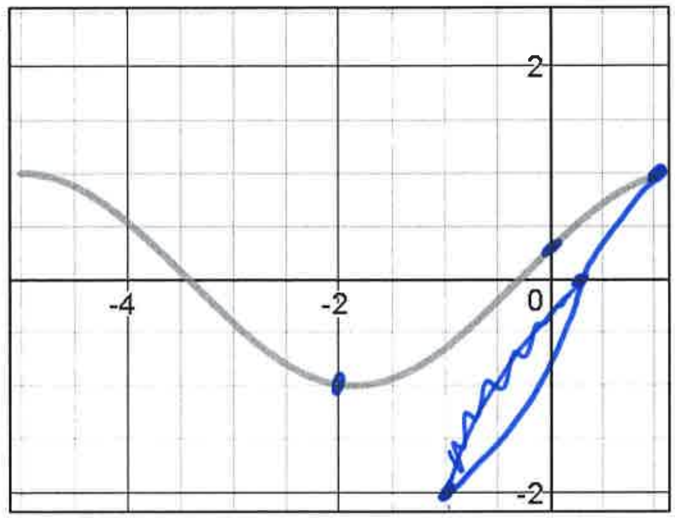


Invertible? No (fails HLT)

Restricted Domain? [-2, 2]

(-2,-3) (0,-2) (2,-1)

2.

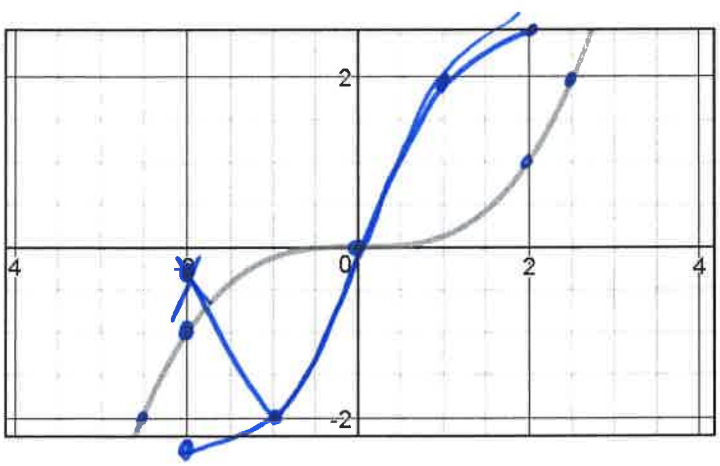


Invertible? NO

Restricted Domain? [-2, 1]

(-2,-1) (0, 0.5) (1, 1)

3.

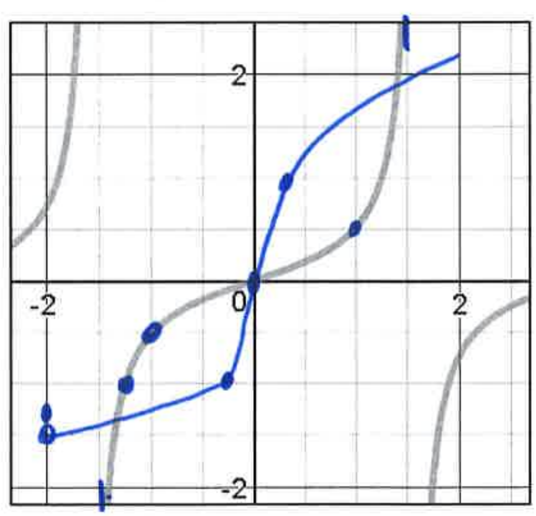


Invertible? yes!

Restricted Domain? NA

(-2.5, -2) (-2,-1) (0,0) (2,1) (2.5,2)

4.



Invertible? NO

Restricted Domain? [-1.5, 1.5]

(-1.5,-2)(-1,-0.5) (0,0) (1,0.5)

For Exercises 5-6, find an equation for $f^{-1}(x)$.

5. $f(x) = 4 - \frac{3}{2}x$ $\quad f^{-1}(x) = \frac{-2x+8}{3}$

$x = 4 - \frac{3}{2}y$

$x - 4 = -\frac{3}{2}y$

$-\frac{2}{3}(x-4) = y$

6. $f(x) = 2x^3 + 3$ $\quad f^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$

$x = 2y^3 + 3$

$x - 3 = 2y^3$

$\frac{x-3}{2} = y^3$

For Exercises 7-8, determine whether the functions are inverses of one another.

7. $f(x) = \frac{-16+x}{4}$ $\quad g(x) = 4x + 16$

$x = \frac{-16+y}{4}$

$4x = -16 + y$

$4x + 16 = y$

yes, inverses

8. $f(x) = \frac{4}{-x-2} + 2$ $\quad g(x) = -\frac{1}{x+3}$

Not inverses

$x = -\frac{1}{y+3}$

$x(y+3) = -1$

$y + 3 = \frac{-1}{x}$

$y = \frac{-1}{x} - 3$

For Exercises 9-10, use the given information to find each value.

9. Use the table to answer the questions.

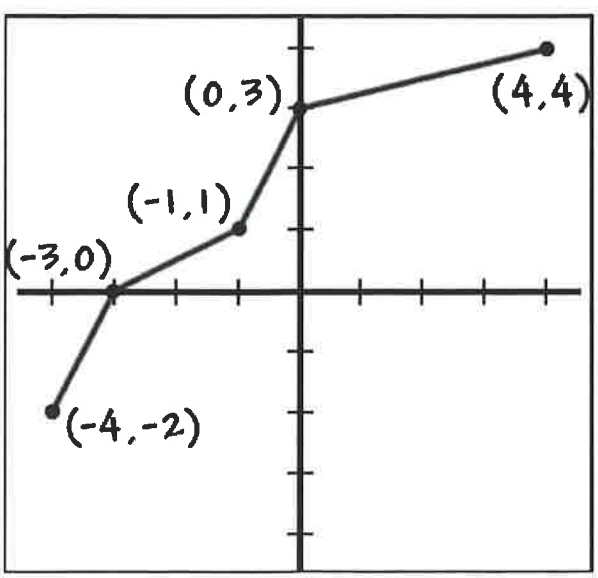
x	$f(x)$
-4	0
-1	2
0	5
5	7

a) $f^{-1}(5) =$ 0

b) $f(0) =$ 5

c) $f(f^{-1}(2)) =$ 2

10. Use the graph to answer the questions.



a) $f^{-1}(f(0)) =$ 0

b) $f^{-1}(4) =$ 4

c) $f(f(-3)) =$ 3