Simple Harmonic Motion

Students will be able to model simple harmonic motion

HPC/RPC

Simple Harmonic Motion: Consistent repetitive motion

Examples: pendulum, weights on a spring, an object bobbing on a wave

*Could have c/d also

$d = \alpha \sin(\omega t + c) + d$

alpha omega

Represented by the equation:

$d = \underset{\text{"a"}}{\alpha} \sin \underset{\text{"b"}}{\omega} t$ or $d = \alpha \cos \omega t$

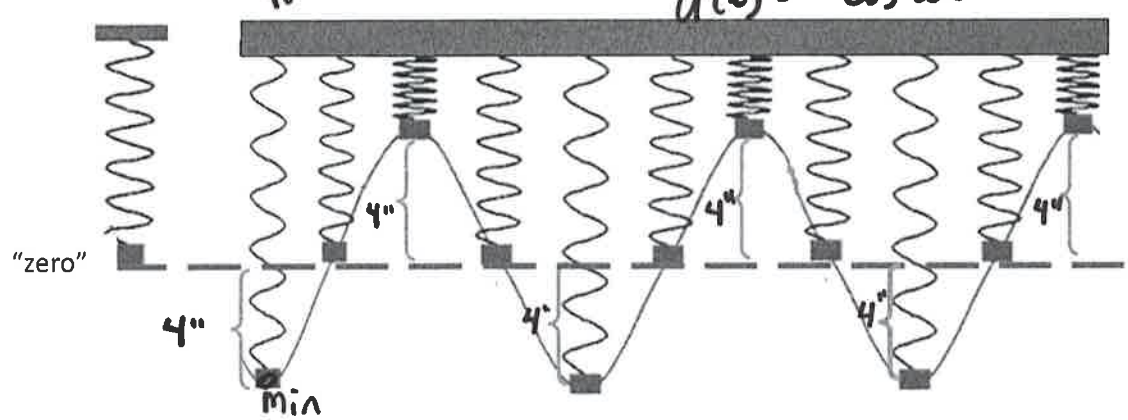
$Period = \frac{2\pi}{\omega}$ $Frequency = \frac{\omega}{2\pi}$ ω = **Angular Velocity**

Example 1:

A block is attached to a spring hung from the ceiling. You pull the block down 4 inches and release it. It takes 5 seconds for the block to complete a cycle. Write an equation that will model the motion of the block.

period

$d(t) = -\cos \omega t$



amp = 4 in.

$\text{period} = \frac{5s}{1} = \frac{2\pi}{\omega}$

$\frac{5\omega}{5} = \frac{2\pi}{5}$

$\omega = \frac{2\pi}{5} = 0.4\pi$

$d(t) = -4\cos 0.4\pi t$

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Example 2:

A point on the end of a tuning fork moves in simple harmonic motion by $d = a \sin \omega t$. Find $\boxed{\omega \text{ given}}$ that the tuning fork for the middle C has a frequency of 264 vibrations per second.

ω = Velocity

$$\text{freq.} = \frac{264v}{1s} = \frac{\omega}{2\pi}$$

$$\frac{2\pi \cdot 264v}{1s} = \omega$$

$$\frac{528\pi v}{s} = \boxed{\frac{1659v}{s}}$$

Simple Harmonic Motion Problems

1. A mass on a spring oscillates back and forth and completes one cycle in 0.2 seconds. Its maximum displacement is 9 cm. Write an equation that models this motion.

Period: 0.2 s

a = 9 cm

$\frac{0.2}{1} = \frac{2\pi}{\omega}$

$0.2\omega = 2\pi$

$\omega = 10\pi$

$d = -9\cos 10\pi t$

2. A buoy oscillates in simple harmonic motion as waves go past. It is noted that the buoy moves a $\boxed{\text{total of 3.5 feet}}$ from its low point to its high point, and that it returns to its high point every 10 seconds. Write an equation that describes the motion of the buoy if its high point is at t = 0.

$\text{amp} = \frac{3.5}{2} = 1.75\text{ ft.}$

$\text{per.}: \frac{10\text{s}}{1} = \frac{2\pi}{\omega}$

$\frac{10\omega}{10} = \frac{2\pi}{10}$

$\omega = 0.2\pi$

$d(t) = 1.75\cos 0.2\pi t$

3. Jacquelyn and Aaron decide to ride the Ferris wheel at the local carnival. The wheel had a 20 foot diameter and turns at 4 rpm with its lowers point 4 feet above the ground. Assume that their height, h, above the ground is a sinusoidal function of time t, where t = 0 represents the lowest point of the wheel.

d=20

10ft.

Intercept

10ft.

4ft.

$d = -10\cos 8\pi t + 14$

$\text{freq.} = \frac{4\text{ rot.}}{1\text{ min}}$

$\frac{4}{1} = \frac{\omega}{2\pi}$

$8\pi = \omega$

4. A point on the tip of a tuning fork vibrates in harmonic motion described by $d = 14\sin \omega t$. $\boxed{\text{Find } \omega}$ for a tuning fork that has a frequency of 528 vibrations per second.

$$\text{freq.} = \frac{528v}{1 \text{ sec.}} = \frac{\omega}{2\pi}$$

$$528 \cdot 2\pi = \omega$$

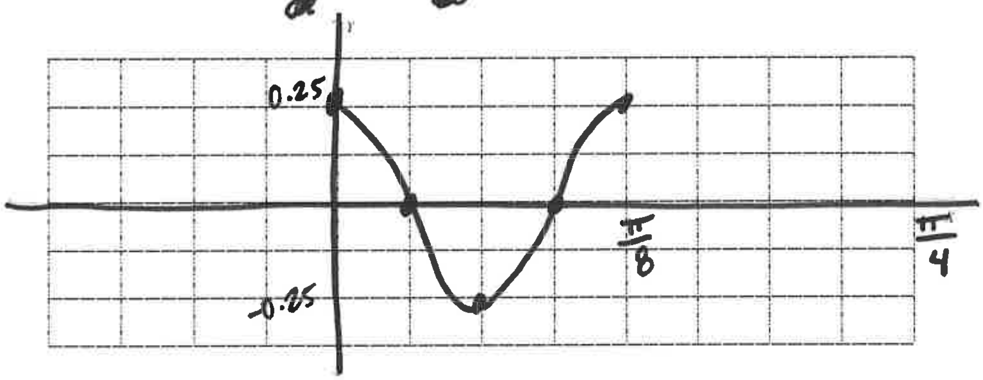
$$1056\pi = \omega \approx 3317.52\ \omega$$

5. A ball that is bobbing up and down on the end of a spring is modeled by the equation $y = \frac{1}{4}\cos 16t$, $for\ t > 0$, where y is in feet and t is in seconds

~~[illegible] displacement of 3in.~~

a. Graph the function. $a = 1/4 \quad b = 16$

period: $\frac{2\pi}{16} = \frac{\pi}{8}$



b. What is the period of the oscillations?

$$\frac{2\pi}{16} = \frac{\pi}{8}$$

c. Determine the first time the weight passes the point of equilibrium (y = 0)

$$\frac{\pi}{32}$$