

# Exponential/Logarithmic Graphing

## Pre-Calculus Notes

Name KEY

Period \_\_\_\_\_

When graphing transformations of exponential functions, use the key features (x- and/or y-intercepts and asymptotes) to help you.

Exponential – always has a horizontal asymptote, which is affected by up and down movement. (Also look for y-intercept)

Logarithmic – always has a Vertical asymptote, which is affected by left and right movement. (Also look for x-intercept)

**Example 1:** Describe how to transform  $f(x) = e^x$  into the graphs of the given functions, then graph:

a.  $g(x) = 3e^x$  (Red)

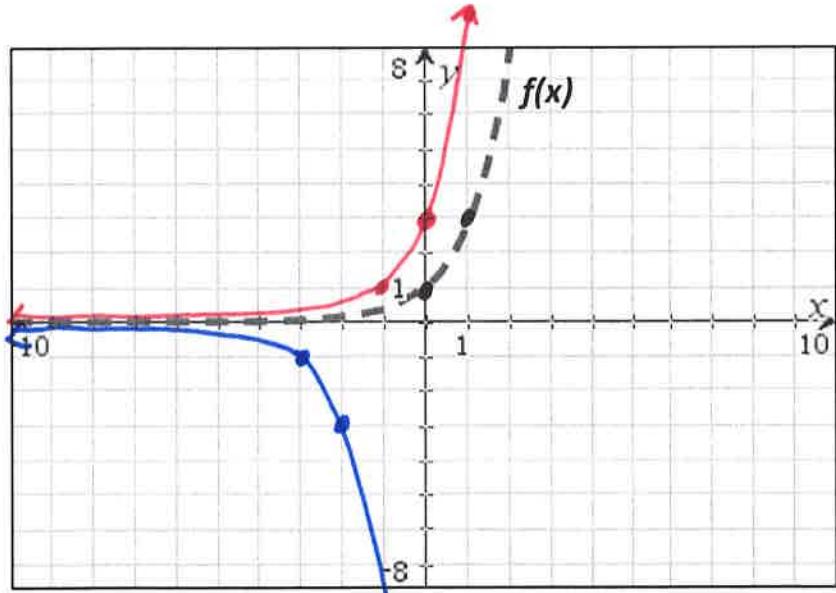
Vertical stretch by factor of 3

$$(x, y) \rightarrow (x, 3y)$$

$$(0, 1) \rightarrow (0, 3)$$

$$(1, 3) \rightarrow (1, 9)$$

$$(-1, \frac{1}{3}) \rightarrow (-1, 1)$$



b.  $k(x) = -3e^{x+2}$  (Blue)

Reflect over x-axis

Vertical stretch by 3

Left 2

$$(x, y) \rightarrow (x-2, -3y)$$

$$(0, 1) \rightarrow (-2, -3)$$

$$(1, 3) \rightarrow (-1, -9)$$

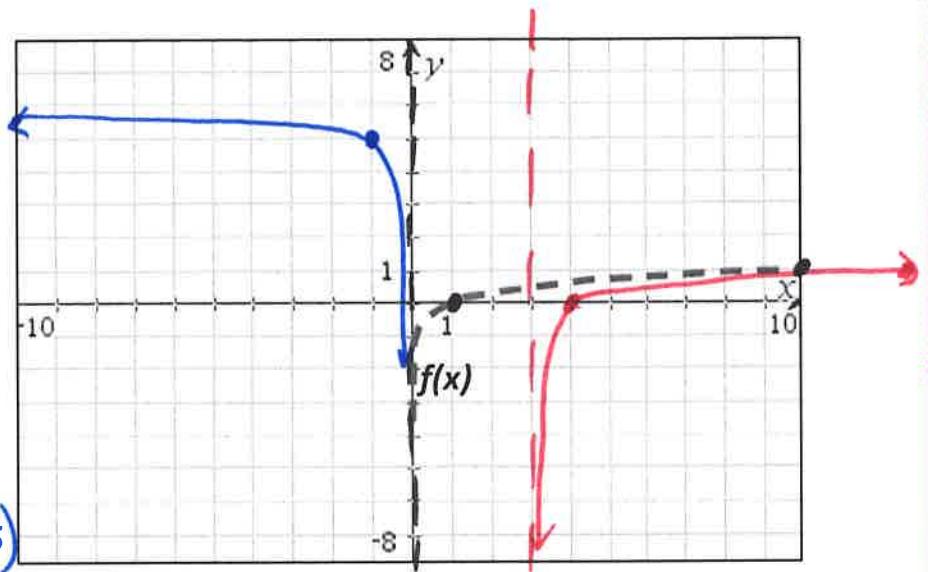
$$(-1, \frac{1}{3}) \rightarrow (-3, -1)$$

**Example 2:** Describe how to transform  $f(x) = \log x$  into the graphs of the given functions:

a.  $g(x) = -\log(x - 3)$  (Red)

Reflect over  $x$ -axis  
Right 3 → (asymptote moves)  
 $(x, y) \rightarrow (x+3, -y)$   
 $(1, 0) \rightarrow (4, 0)$   
 $(10, 1) \rightarrow (13, 1)$

(Blue)  
b.  $h(x) = 0.5 \log(-x) + 5$   
Vertical shrink by 0.5  
Reflect over  $y$ -axis  
up 5  
 $(x, y) \rightarrow (-x, 0.5y + 5)$  or  $(-x, \frac{y}{2} + 5)$   
 $(1, 0) \rightarrow (-1, 5)$   
 $(10, 1) \rightarrow (-10, 5.5)$



**Example 3:** Given the graph of the original function,  $f(x)$ , graph the transformed function and analyze it for asymptotes, intercepts and end behavior.

a)  $g(x) = -3(4^x) + 2$  (asymptote moves)

Reflect over  $x$ -axis  
Vertical stretch by 3  
up 2  
 $(x, y) \rightarrow (x, -3y + 2)$   
 $(0, 1) \rightarrow (0, -1)$   
 $(-1, 1/4) \rightarrow (-1, 1.25)$   
 $(1, 4) \rightarrow (1, -10)$

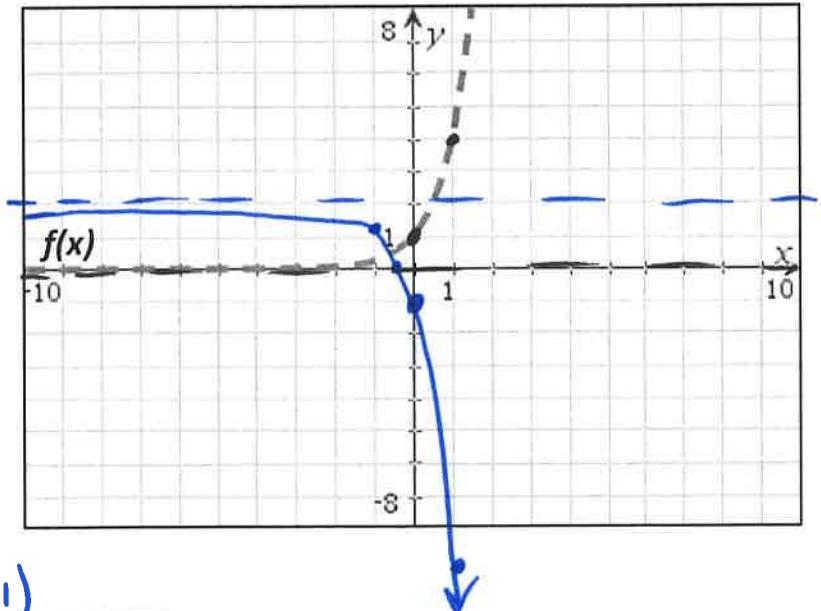
hard to see,  
estimate  
x-intercept  $(-0.2, 0)$

asymptote  $y = 2$

y-intercept  $(0, -1)$

end behavior: As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow 2$

As  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$



Be careful!  
Reflect + shift = No LIES.

b)  $g(x) = \ln(-x+1)$

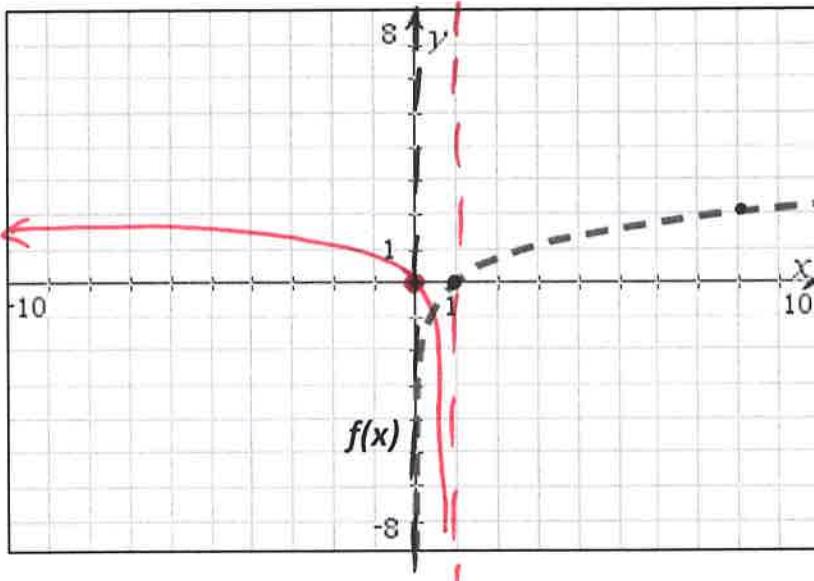
Reflect over y-axis

Right 1 (asymptote at 1)

$$(x,y) \rightarrow (-x+1, y)$$

$$(1,0) \rightarrow (0,0)$$

↑ this is really  
the only clear  
point for  $\ln(x)$  ... so estimate.



x-intercept (0,0)

y-intercept (0,0)

asymptote  $x=1$

end behavior: As  $x \rightarrow -\infty, f(x) \rightarrow \infty$

As  $x \rightarrow 1, f(x) \rightarrow -\infty$

c)  $g(x) = 2^{2x} - 3$

Horizontal Shrink by 2  
Down 3 (asymptote moves)

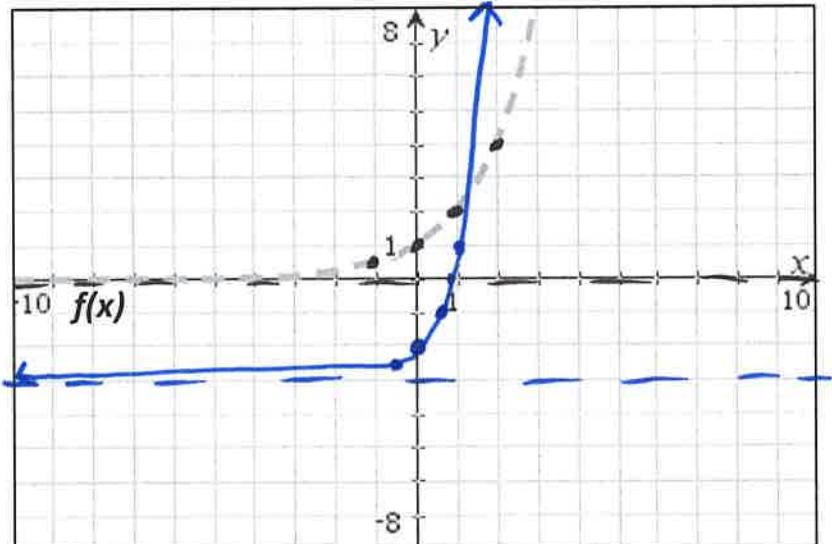
$$(x,y) \rightarrow \left(\frac{x}{2}, y-3\right)$$

$$(0,1) \rightarrow (0,-2)$$

$$(-1, \frac{1}{2}) \rightarrow (-\frac{1}{2}, -2.5)$$

$$(1, 2) \rightarrow (\frac{1}{2}, -1)$$

$$(2, 4) \rightarrow (1, 1)$$



Hard to see  
x-intercept (0.8, 0)

y-intercept (0, -2)

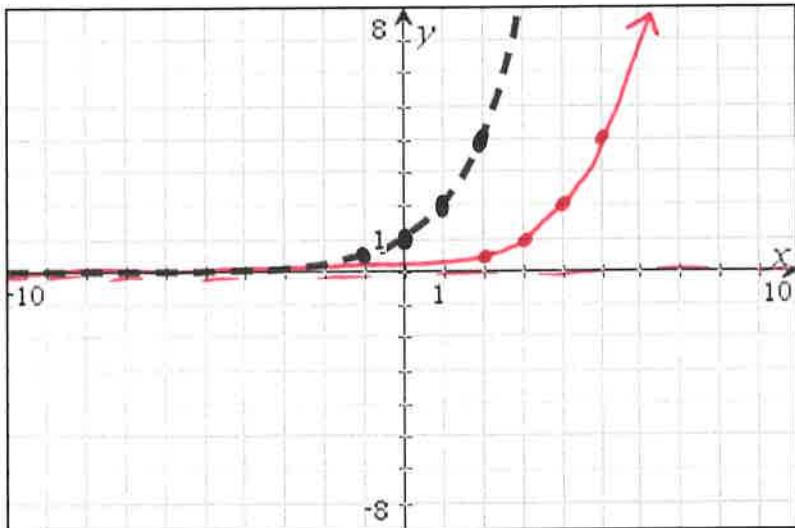
asymptote  $y = -3$

end behavior: As  $x \rightarrow -\infty, f(x) \rightarrow -3$

As  $x \rightarrow \infty, f(x) \rightarrow \infty$

Sketch the graph of  $g(x)$  onto the coordinate grid (which shows the graph of  $f(x)$  already) using specific points. Also, analyze the graph for asymptotes, intercepts and end behavior.

1.  $f(x) = 2^x$ ,  $g(x) = 2^{x-3}$  Right 3  $(x,y) \rightarrow (x+3, y)$



x-intercept None  
y-intercept  $(0, 0.2)$  → guess

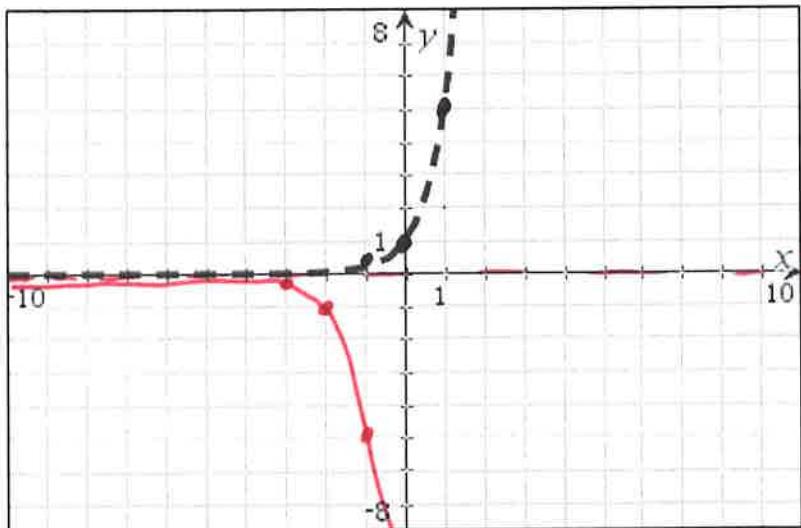
asymptote  $y = 0$

end behavior:

As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow 0$

As  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$

2.  $f(x) = 5^x$ ,  $g(x) = -5^{x+2}$  Reflect over x-axis, left 2  $(x,y) \rightarrow (x-2, -y)$



x-intercept None  
y-intercept  $(0, -25)$  Can't see... but  $-5^{(0+2)} = -5^2 = -25$

asymptote  $y = 0$

end behavior:

As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow 0$

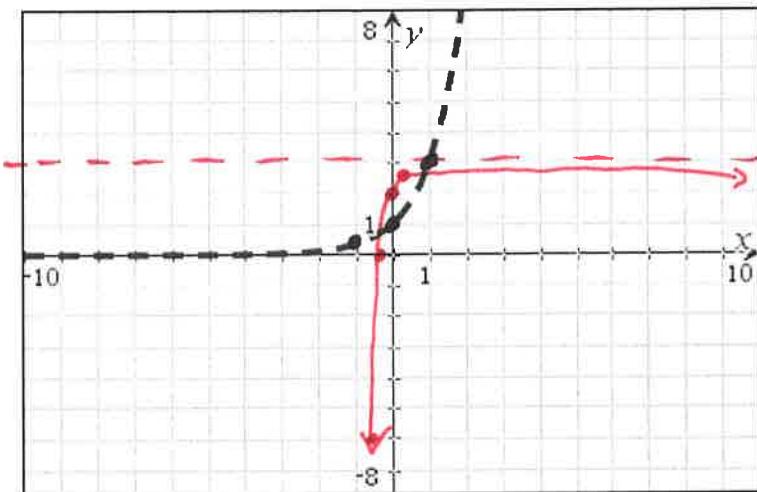
As  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$

$(-1, 1/5) \rightarrow (-3, -1/5)$

$(0, 1) \rightarrow (-2, -1)$

$(1, 5) \rightarrow (-1, -5)$

3.  $f(x) = 3^x$ ,  $g(x) = -3^{-3x} + 3$  Reflect X Reflect Y Horiz. shrink by 3 up 3  $(x, y) \rightarrow \left(-\frac{x}{3}, -y + 3\right)$



x-intercept  $\underline{-\frac{1}{3}}$

y-intercept  $\underline{2}$

asymptote  $\underline{y = 3}$

end behavior:

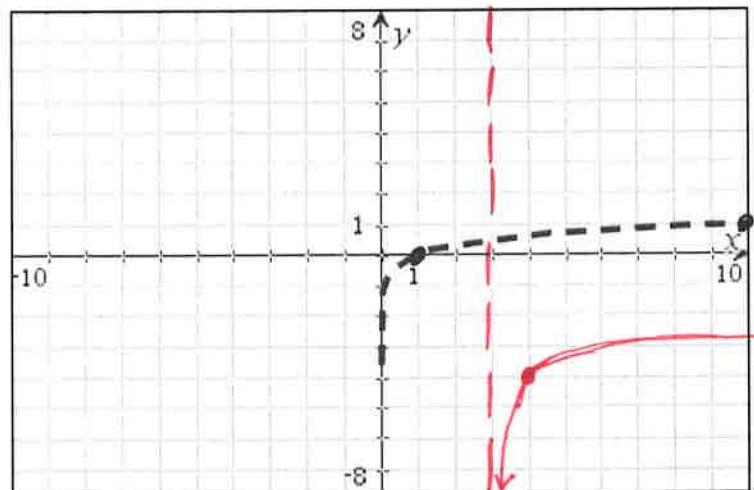
As  $x \rightarrow \underline{-\infty}$ ,  $f(x) \rightarrow \underline{-\infty}$

As  $x \rightarrow \underline{\infty}$ ,  $f(x) \rightarrow \underline{3}$

$$\begin{aligned} (-1, \frac{1}{3}) &\rightarrow (1/3, 2.6) & (2, 9) &\rightarrow \left(\frac{2}{3}, -6\right) \\ (0, 1) &\rightarrow (0, 2) \\ (1, 3) &\rightarrow (-1/3, 0) \end{aligned}$$

4.  $f(x) = \log(x)$ ,  $g(x) = \log(x - 3) - 4$  Right 3, down 4

$$(x, y) \rightarrow (x+3, y-4)$$



x-intercept ~~can't see~~

y-intercept None

asymptote  $\underline{x = 3}$

end behavior:

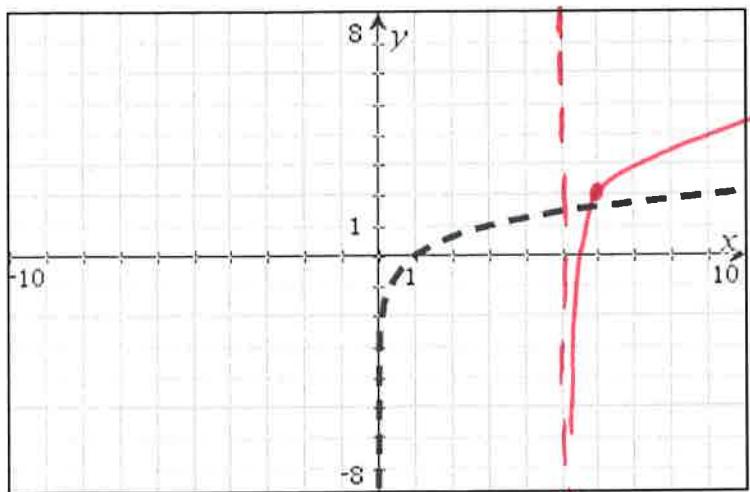
As  $x \rightarrow \underline{3}$ ,  $f(x) \rightarrow \underline{-\infty}$

As  $x \rightarrow \underline{\infty}$ ,  $f(x) \rightarrow \underline{\infty}$

$$(1, 0) \rightarrow (4, -4)$$

$$(10, 1) \rightarrow (13, -3)$$

5.  $f(x) = \log_3 x$ ,  $g(x) = 3 \log_3(x - 5) + 2$  vertical stretch by 3, Right 5, up 2  $(x, y) \rightarrow (x+5, 3y+2)$



x-intercept  $\underline{(5.5, 0)}$

y-intercept None

asymptote  $\underline{x = 5}$

end behavior:

As  $x \rightarrow \underline{5}$ ,  $f(x) \rightarrow \underline{-\infty}$

As  $x \rightarrow \underline{\infty}$ ,  $f(x) \rightarrow \underline{\infty}$

$$(1, 0) \rightarrow (6, 2)$$

$$(10, 1) \rightarrow (15, 5)$$

\*New graph\*