DOT PRODUCT & WORK Day 3 **Name:** Key

Find the angle between **v** *and* **w**. *Round to the nearest tenth of a degree. Determine if the angles are parallel, orthogonal, or neither.*

1. a. $v = \langle 2, -1 \rangle$ $w = \langle 3, 4 \rangle$

$v \cdot w = (2)(3) + (-1)(4) = 2$

$\|v\| = \sqrt{2^2+(-1)^2} = \sqrt{5}$

$\|w\| = \sqrt{3^2+4^2} = 5$

$\theta = \cos^{-1}\left(\frac{2}{5\sqrt{5}}\right) = 79.7°$ Neither

2. a. $v = \langle -2, 5 \rangle$ $w = \langle 4, 3 \rangle$

$v \cdot w = (-2)(4) + (5)(3) = 7$

$\|v\| = \sqrt{(-2)^2+(5)^2} = \sqrt{29}$

$\|w\| = \sqrt{4^2+3^2} = 5$

$\theta = \cos^{-1}\left(\frac{7}{5\sqrt{29}}\right) = 74.9°$ Neither

Find $\text{proj}_w v$.

3. a. $v = \langle 3, -2 \rangle$ $w = \langle 1, -1 \rangle$

$v \cdot w = 3(1) + (-2)(-1) = 5$

$\|w\| = \sqrt{1^2+1^2} = \sqrt{2}$

$\text{Proj}_w v = \frac{5}{(\sqrt{2})^2}\langle 1, -1 \rangle = \langle \frac{5}{2}, -\frac{5}{2} \rangle$

4. a. $v = \langle 2, 4 \rangle$ $w = \langle -3, 6 \rangle$

$v \cdot w = (2)(-3) + (4)(6) = 18$

$\|w\| = \sqrt{(-3)^2+(6)^2} = \sqrt{45}$

$|\text{Proj}_w v| = \frac{18}{(\sqrt{45})^2}\langle -3, 6 \rangle = \langle -\frac{6}{5}, \frac{12}{5} \rangle$

b. $v = \langle 1, -1 \rangle$ $w = \langle -2, 2 \rangle$

$v \cdot w = (1)(-2) + (-1)(2) = -4$

$\|v\| = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$\|w\| = \sqrt{(-2)^2+(2)^2} = \sqrt{8}$

$\theta = \cos^{-1}\left(\frac{-4}{\sqrt{2}\sqrt{8}}\right) = \cos^{-1}(-1) = 180°$ parallel opp dir.

b. $v = \langle -8, 3 \rangle$ $w = \langle -3, 8 \rangle$

$v \cdot w = (-8)(-3) + (3)(8) = 48$

$\|v\| = \sqrt{(-8)^2+(3)^2} = \sqrt{73}$

$\|w\| = \sqrt{(-3)^2+(8)^2} = \sqrt{73}$

$\theta = \cos^{-1}\left(\frac{48}{73}\right) = 48.89°$ Neither

b. $v = \langle 3, -2 \rangle$ $w = \langle 2, 1 \rangle$

$v \cdot w = (3)(2) + (-2)(1) = 6 - 2 = 4$

$\|w\| = \sqrt{2^2+1^2} = \sqrt{5}$

$\text{Proj}_w v = \frac{4}{(\sqrt{5})^2}\langle 2, 1 \rangle = \langle \frac{8}{5}, \frac{1}{5} \rangle$

4. a. $v = \langle 2, 4 \rangle$ $w = \langle -3, 6 \rangle$

$v \cdot w = (2)(-3) + (4)(6) = 18$

$\|w\| = \sqrt{(-3)^2+(6)^2} = \sqrt{45}$

$|\text{Proj}_w v| = \frac{18}{(\sqrt{45})^2}\langle -3, 6 \rangle = \langle -\frac{6}{5}, \frac{12}{5} \rangle$

b. $v = \langle 1, 2 \rangle$ $w = \langle 3, 6 \rangle$

$v \cdot w = (1)(3) + (2)(6) = 15$

$\|w\| = \sqrt{45}$

$|\text{proj}_w v| = \frac{15}{(\sqrt{45})^2}\langle 3, 6 \rangle = \langle 1, 2 \rangle$

Solve the following problems.

5. The components of $v = \langle 240, 300 \rangle$ represent the respective number of gallons of regular and premium gas sold at a station. The components of $w = \langle 2.90, 3.07 \rangle$ represent the respective prices per gallon for each kind of gas. Find **v•w** and describe what the answer means in practical terms.

$v \cdot w = (240)(2.90) + (300)(3.07)$

$= \$501.60 + 921$ $\$1617.00$

$= 1422.00 \rightarrow$ Represent Total Sales of both fuels.

6. The components of $v = \langle 180, 450 \rangle$ represent the respective number of one-day and three-day videos rented from a video store. The components of $w = \langle 3, 2 \rangle$ represent the prices to rent the one-day and three-day videos. Find **v•w** and describe what the answer means in practical terms.

$v \cdot w = (180)(3) + (450)(2)$

$= 540 + 900$

$= 1440.00 \rightarrow$ Represent total sales of both one- and three-day rentals

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7. Find the work done when a crane lifts a 6000 pound boulder through a vertical distance of 12 feet. Round to the nearest foot-pound.

Work = $6000 \cdot \cos 0^\circ \cdot 12$

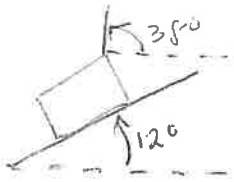
$W = 72{,}000$ foot-pounds

8. A wagon is pulled along level ground by exerting a force of 25 pounds on a handle that makes a 38° with the horizontal. How much work is done pulling the wagon 100 feet? Round to the nearest foot pound.

Work: $25 \cdot \cos 38^\circ \cdot 100$

$W = 1970$ ft. pounds

9. A force of 60 pounds on a rope is used to pull a box up a ramp inclined at 12° from the horizontal. The rope forms an angle of 38° with the horizontal. How much work is done pulling the box 20 feet along the ramp?

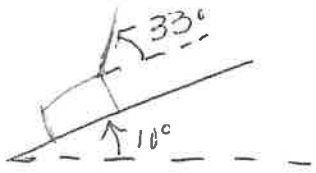


$W = 60 \cdot \cos 26^\circ \cdot 20$

$W = 1079$ ft. lbs.

$\theta = 38 - 12 = 26^\circ$

10. A force of 80 pounds on a rope is used to pull a box up a ramp inclined at 10° from the horizontal. The rope forms an angle of 33° with the horizontal. How much work is done pulling the box 25 feet along the ramp?



$W = 80 \cdot \cos 23^\circ \cdot 25$

$W = 1841$ ft. lbs.

$\theta = 33 - 10$