Completing the Square

What is the purpose of completing the square?
The purpose of completing the square is to factor and solve a prime quadratic equation or to more easily graph a conic section.

When an equation does not contain a perfect square trinomial, you will sometimes need to add a term to make the trinomial a perfect square in order to solve it.

Consider a trinomial in general form: $Ax^2 + Bx + C = 0$

The steps to make a trinomial a perfect square are listed below:

Step 1: Divide the equation by A so that the leading coefficient is 1

Step 2: Subtract C from both sides of the equation

Step 3: Divide B by 2 and square the result. Then add to both sides of the equation

Step 4: Factor the left side of the equation.

Step 5: Take the square root of both sides of the equation.

Step 6: Solve for x

Example 1: Solve the following quadratic equations by finding the square root.

a. $x^2 - 8x + 16 = 25$

$\sqrt{(x-4)^2} = \sqrt{25}$

$x - 4 = \pm 5$

$x - 4 = 5$ or $x - 4 = -5$

$x = 9$ $\quad x = -1$

b. $2x^2 - 24x + 8 = 0$

$2(x^2 - 12x + 4) = 0$

$(x^2 - 12x + 36) + 4 = 0 + 36$

$(x-6)^2 + 4 = 36$

$\sqrt{(x-6)^2} = \sqrt{32}$

$x - 6 = \pm\sqrt{32} = \pm 4\sqrt{2}$

$x = 6 \pm 4\sqrt{2}$

① $2(x^2 - 12x + 4) = 0$

② $\cancel{2}(x^2 - 12x) = -4$

$\frac{-12}{2} = -6^2 = 36$ ③ $x^2 - 12x + 36 = -4 + 36$

④ $(x-6)^2 = 32$

⑤ $x - 6 = \pm\sqrt{32}$

⑥ $x = 6 \pm 4\sqrt{2}$

32

8 4

2 4

Your Turn!!! :)

a. $x^2 - 8x - 128 = 0$

$(x^2 - 8x + 16) - 128 = 0 + 16$

$(x-4)^2 = 144$

$x - 4 = \pm 12$

$x = 16$ or $x = -8$

$-\frac{8}{2} = 4^2 = 16$

$x^2 - 8x = 128$

$x^2 - 8x + 16 = 128 + 16$

$(x-4)^2 = 144$

$x - 4 = \pm 12$

$x = 16$ or $x = -8$

b. $3x^2 - 36x - 144 = 0$

$3(x^2 - 12x - 48) = 0$

$(x^2 - 12x + 36) - 48 = 0 + 36$

$(x-6)^2 = 84$

$x - 6 = \pm\sqrt{84}$

$x = 6 \pm \sqrt{84}$

$\cancel{3}(x^2 - 12x - 48) = 0$

$x^2 - 12x = 48$

$x^2 - 12x + 36 = 48 + 36$

$(x-6)^2 = 84$

$x - 6 = \pm\sqrt{84}$

$x = 6 \pm \sqrt{84}$

$6 \pm 2\sqrt{21}$

84

42 2

21 2

*Adding 1 to the x^2-2x, but b/c of the 4 I have to add 4 on the right.

Conic Sections are often written in the general form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. In order to write them in the more "user-friendly" Standard Form, you must complete the square to create Perfect Square Trinomials for both x and y.

Example 2: Write the equation in Standard Form for each Conic Section. $\frac{x^2}{?} + \frac{y^2}{?} = 1$

a. $4x^2 + y^2 - 8x - 8 = 0$

$4x^2 - 8x + y^2 = 8$

$4(x^2-2x+1) + y^2 = 8+4$

$\frac{4(x-1)^2}{12} + \frac{y^2}{12} = \frac{12}{12}$

$\frac{(x-1)^2}{3} + \frac{y^2}{12} = 1$

$\frac{-2}{2} = 1^2 = 1$

$4x^2 - 8x + y^2 = 8$

$4(x^2-2x) + y^2 = 8$

$4(x^2-2x+1) + y^2 = 8+4$

$\frac{4(x-1)^2}{12} + \frac{y^2}{12} = \frac{12}{12}$

$\frac{(x-1)^2}{3} + \frac{y^2}{12} = 1$

b. $16x^2 - 9y^2 - 96x + 36y - 36 = 0$

$16x^2 - 96x - 9y^2 + 36y = 36$

$16(x^2-6x+9) - 9(y^2-4y+4) = 36 + 144 - \cancel{36}$

$\frac{16(x-3)^2}{144} - \frac{9(y-2)^2}{144} = \frac{144}{144}$

$\frac{(x-3)^2}{9} - \frac{(y-2)^2}{16} = 1$

$-\frac{6}{2} = 3^2 = 9$ $\frac{4}{2} = 2^2 = 4$

$16x^2 - 96x - 9y^2 + 36y = 36$

$16(x^2-6x) - 9(y^2-4y) = 36$

$16(x^2-6x+9) - 9(y^2-4y+4) = 36 + (16\times9) - 9(4)$

$\frac{16(x-3)^2}{144} - \frac{9(y-2)^2}{144} = \frac{144}{144}$

$\frac{(x-3)^2}{9} - \frac{(y-2)^2}{16} = 1$

<u>Example 2 (continued):</u> **Write the equation in Standard Form for each Conic Section.**

c. $y^2 - 4y - 2x + 6 = 0$

SIMPLIFY & SOLVE FOR x

d. $x^2 - 16x - 8y + 80 = 0$

$$\underbrace{(y^2-4y+4)}-2x+6-4=0$$

$\cup$ $\cap$

$$y=a(x-h)^2+k$$

(h,k) VERTEX

$$(y-2)^2-2x+2=0$$

$$(y-2)^2+2=2x$$

$$\frac{(y-2)^2}{2}+1=x \rightarrow x=\frac{1}{2}(y-2)^2+1$$

Precalculus

Name________________________ ID: 1

General to Standard Form of Conic Sections

Date________________ Period____

Use the information provided to write the standard form equation of the conic section.

1) $x^2 + 4y^2 - 20x + 80y + 400 = 0$

$$\boxed{\frac{(x-10)^2}{100} + \frac{(y+10)^2}{25} = 1}$$

$x^2 - 20x + 4y^2 + 80y = -400$

$x^2 - 20x + 100 + 4(y^2 + 20y + 100) = -400 + 100 + 400$

$\frac{(x-10)^2}{100} + \frac{4(y+10)^2}{100} = \frac{100}{100}$

2) $-x^2 + y^2 - 10x + 14y - 12 = 0$

$$\boxed{\frac{(y+7)^2}{36} - \frac{(x+5)^2}{36} = 1}$$

$-1(x^2 + 10x) + y^2 + 14y = 12$

$-1(x^2 + 10x + 25) + y^2 + 14y + 49 = 12 + (-25) + 49$

$\frac{(x+5)^2}{36} + \frac{(y+7)^2}{36} = \frac{36}{36}$

3) $x^2 + 4y^2 - 16x + 8y - 76 = 0$

$$\boxed{\frac{(x-8)^2}{144} + \frac{(y+1)^2}{36} = 1}$$

$x^2 - 16x + 4(y2 + 2y) = 76$

$x^2 - 16x + 64 + 4(y^2 + 2y + 1) = 76 + 64 + 4$

$\frac{(x-8)^2}{144} + \frac{4(y+1)^2}{144} = \frac{144}{144}$

4) $4x^2 + y^2 - 64x + 6y + 249 = 0$

$$\boxed{\frac{(x-8)^2}{4} + \frac{(y+3)^2}{16} = 1}$$

$4(x^2 - 16x) + y^2 + 6y = -249$

$4(x^2 - 16x + 64) + y^2 + 6y + 9 = -249 + 256 + 9$

$\frac{4(x-8)^2}{16} + \frac{(y+3)^2}{16} = \frac{16}{16}$

~~5)~~ $-4x^2 - 24x + y - 35 = 0$

$y = 4(x+3)^2 - 1$

6) $-4x^2 + 9y^2 - 72x + 144y - 324 = 0$

$$\boxed{\frac{(y+8)^2}{64} - \frac{(x+9)^2}{144} = 1}$$

$-4(x^2 + 18x) + 9(y^2 + 16y) = 324$

$-4(x^2 + 18x + 81) + 9(y^2 + 16y + 64) = 324 - 324 + 576$

$\frac{-4(x+9)^2}{576} + \frac{9(y+8)^2}{576} = \frac{576}{576}$

7) $-x^2 + y^2 - 8x - 4y - 76 = 0$

$$\boxed{\frac{(y-2)^2}{64} - \frac{(x+4)^2}{64} = 1}$$

$-1(x^2 + 8x) + y^2 - 4y = 76$

$-1(x^2 + 8x + 16) + y^2 - 4y + 4 = 76 - 16 + 4$

$\frac{-1(x+4)^2}{64} + \frac{(y-2)^2}{64} = \frac{64}{64}$

~~8)~~ $x^2 - 12x + 4y + 56 = 0$

$y = -\frac{1}{4}(x-6)^2 - 5$

9) $x^2 + 6x + 12y + 45 = 0$

$y = -\frac{1}{12}(x+3)^2 - 3$

10) $x^2 - 4y^2 - 10x + 56y - 315 = 0$

$\boxed{\frac{(x-5)^2}{144} - \frac{(y-7)^2}{36} = 1}$

11) $16x^2 + y^2 - 64x - 14y - 31 = 0$

$\boxed{\frac{(x-2)^2}{9} + \frac{(y-7)^2}{144} = 1}$

12) $4x^2 + 5y^2 + 24x + 30y - 119 = 0$

$\boxed{\frac{(x+3)^2}{50} + \frac{(y+3)^2}{40} = 1}$

13) $x^2 - 4y^2 + 20x + 16y + 68 = 0$

$\frac{(x+10)^2}{16} - \frac{(y-2)^2}{4} = 1$

14) $2x^2 + 32x + y + 131 = 0$

$y = -2(x+8)^2 - 3$

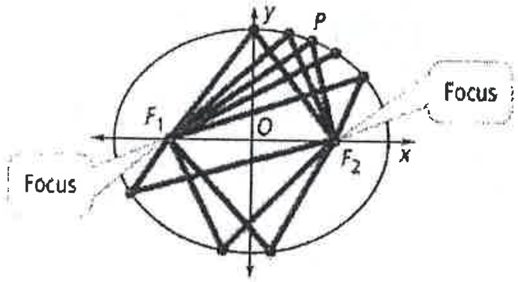
15) $10x^2 - 180x + y + 807 = 0$

$\boxed{y = -10(x-9)^2 + 3}$

  Made with Infinite Precalculus

Ellipses

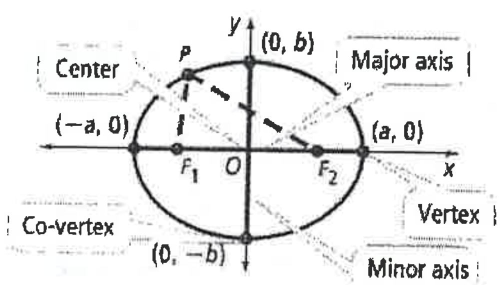
Graph



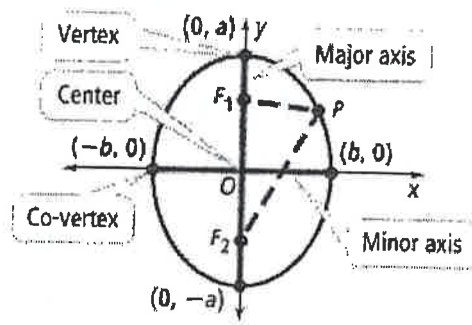
$PF_1 + PF_2 = k$, where $k > F_1F_2$.

Ellipse: a set of all points P in a plane such that the sum of the distances from P to two fixed points F_1 and F_2 is a constant.

Foci of an Ellipse: One of the two fixed points.



Horizontal Ellipse



Vertical Ellipse

Major Axis: The segment that contains the foci and has end points on the ellipse. Longest Axis

Center of the ellipse: The midpoint of the ~~major~~ both axis.
* Where they intersect

Minor Axis: Perpendicular to the major axis the center.
Shortest Axis

Vertices: The endpoints of the major axis

Co-vertices: The endpoints of the minor axis.

The standard form of the equation of an ellipse with center (h,k) is:

(vertices are ±a distance from the center)

$$\boxed{\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1}$$

a^2 is under the major axis

Key Concept Properties of Ellipses with Center (0, 0)

	Horizontal Ellipses	Vertical Ellipses
Standard Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b>0$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1,\ a>b>0$
Major Axis	horizontal	vertical
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Co-vertices	$(0, \pm b)$	$(\pm b, 0)$
Foci	$(\pm c, 0)$ on x-axis	$(0, \pm c)$ on y-axis

The length of the major axis is $2a$ and the length of the minor axis is $2b$.
For any point P on an ellipse, $PF_1 + PF_2 = 2a$.

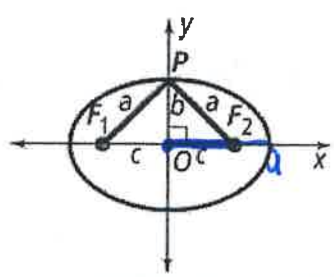
If "x" stays the same then it's a vertical. If "y", then horizontal.

Key Concept *Equations of Ellipses with Centers at (h, k)*

Standard Form of Equation	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
Direction of Major Axis	horizontal	vertical
Foci	$(h \pm c, k)$	$(h, k \pm c)$

or $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

Since the co-vertex $P(0, b)$ is on the ellipse, $PF_1 + PF_2 = 2a$. If you denote the distance from each focus to the center of the ellipse by c, then a, b, and c are the lengths of the sides of a right triangle, as shown in the ellipse at the right. Thus, the distances from the center to each vertex, to each co-vertex, and to each focus are related by the Pythagorean Theorem: $a^2 = b^2 + c^2$.



If $(\pm a, 0)$, $(0, \pm b)$, and $(\pm c, 0)$ are the vertices, the co-vertices, and the foci of an ellipse, respectively,

$c^2 = a^2 - b^2$

Example 11: Write the equation for an ellipse in standard form centered at the origin with vertex (-6,0) and co-vertex (0,3).

a

b

(0,0)

$$\frac{x^2}{36}+\frac{y^2}{9}=1$$

$$\frac{x^2}{6^2}+\frac{y^2}{3^2}=1$$

Example 12: What are the foci of the ellipse with the equation $25x^2+9y^2=225$? Graph the foci and the ellipse.

$$\frac{25x^2}{225}+\frac{9y^2}{225}=\frac{225}{225}$$

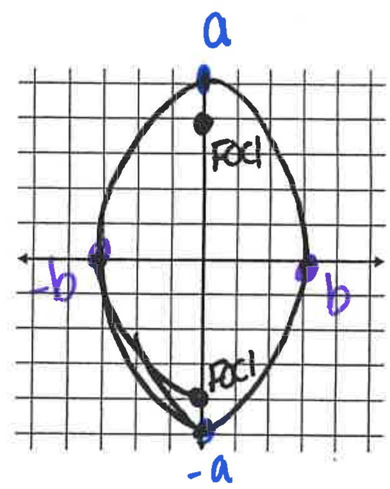
$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

$b:3$ $a:5$

$25>9$ so the major axis is vertical

$c^2=a^2-b^2=25-9=16$ $c=\pm4$

foci are $(0,-4)$ and $(0,4)$



Example 13:

Whispering Gallery A room with an elliptical ceiling (called an *ellipsoid,* since it is 3-dimensional) forms a "whispering gallery." Thanks to the reflective property of the ellipse, a whispered message at one focus can be heard clearly by someone standing across the room at the other focus. If the elliptical ceiling has a major axis of 120 feet and a minor axis of 72 feet, how far apart are the foci?

length of major axis $=2a$

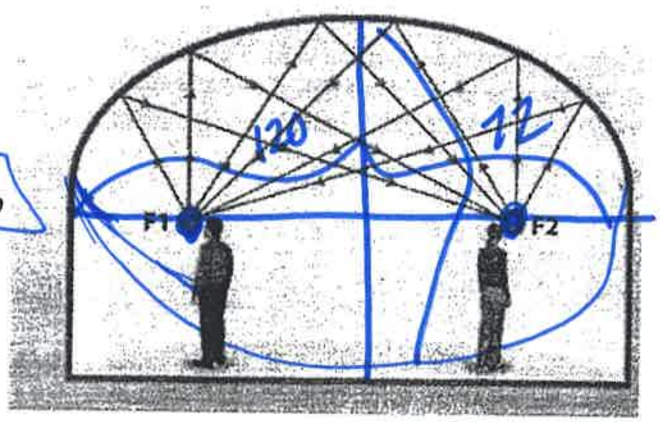
so $2a=120$ $\boxed{a=60}$

minor axis $=2b$ so $2b=72$ $\boxed{b=36}$

$c^2=a^2-b^2=60^2-36^2$

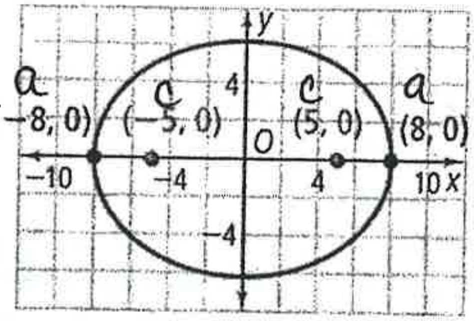
$c=\sqrt{60^2-36^2}=48$

so foci are $2c=96$ ft



Example 14:

What is the standard form equation of the ellipse shown?



$c^2 = a^2 - b^2$

$25 = 64 - b^2$

$b^2 = 64 - 25 = 39$

$\frac{x^2}{64} + \frac{y^2}{39} = 1$

$a^2 \quad b^2$

$c^2 = a^2 - b^2$

$5^2 = 8^2 - b^2$

$25 = 64 - b^2$

$-39 = -b^2$

$\sqrt{39} = \sqrt{b^2}$

Identify the center, vertices, co-vertices, foci, length of the major and minor axes.

1. $\frac{x^2}{64} + \frac{(y-6)^2}{121} = 1$ $a = 11$, $b = 8$ vertical

Center $(0, 6)$

Vertices $(0, 17), (0, -5)$ $(0, 6+11)$ $(0, 6-11)$

Co-vertices $(8, 6)$ $(-8, 6)$ $(0+8, 6)$ $(0-8, 6)$

Foci $(0, 6+\sqrt{57}), (0, 6-\sqrt{57})$

$2(11)$ major axis $= 22$

$2(8)$ minor axis $= 16$

2. $\frac{(x+5)^2}{81} + \frac{(y-1)^2}{144} = 1$ $a = 12$, $b = 9$ vertical

Center $(-5, 1)$

Vertices $(-5, 13), (-5, -11)$ $(-5, 1+12)$ $(-5, 1-12)$

Co-vertices $(4, 1), (-14, 1)$ $(-5+9, 1)$ $(-5-9, 1)$

Foci $(-5, 1+3\sqrt{7}), (-5, 1-3\sqrt{7})$

maj. axis $= 24$

minor axis $= 18$

Identify the center, vertices, co-vertices, foci, length of the major and minor axes and graph the ellipse.

3. $\frac{x^2}{4} + \frac{y^2}{9} = 1$ $a = 3$, $b = 2$

center $(0, 0)$

major axis $= 6$

minor axis $= 4$

vertices $(0, \pm 3)$

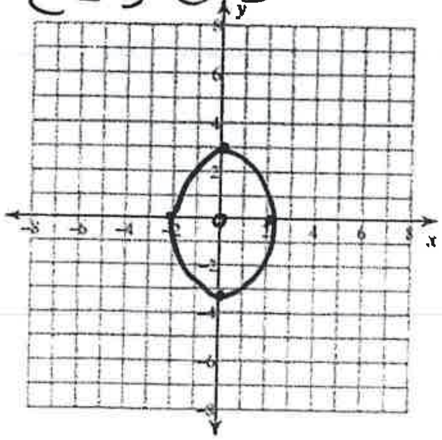
co-vertices $(\pm 2, 0)$

$a^2 = b^2 + c^2$

$9 = 4 + c^2$

$c^2 = 5$ $c = \sqrt{5}$

Foci $(0, \pm\sqrt{5})$

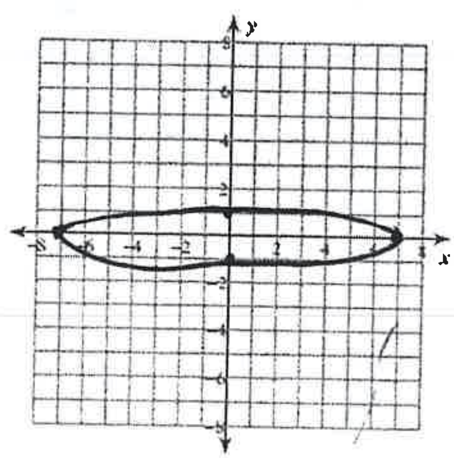


4. $\frac{x^2}{49} + y^2 = 1$ $a = 7$ $b = 1$

Center $(0, 0)$

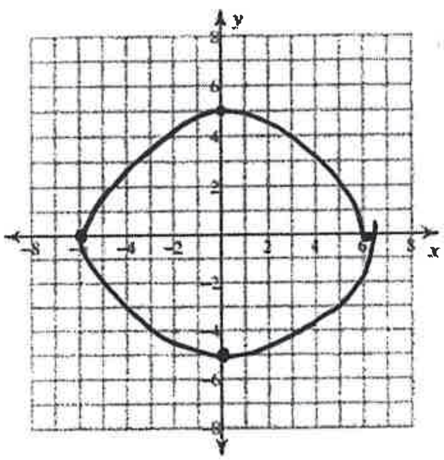
major axis $= 14$

minor axis $= 2$



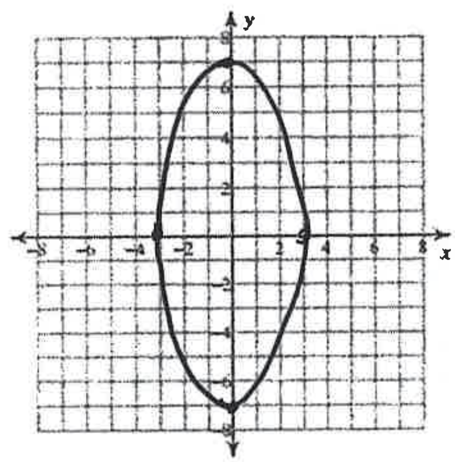
5. $\frac{x^2}{36}+\frac{y^2}{25}=1$ a=6 b=5

Center (0,0)
major axis = 12
minor axis = 10



6. $\frac{x^2}{9}+\frac{y^2}{49}=1$ a=7 b=3

Center (0,0)
major axis = 14
minor axis = 6



Use the information provided to write the standard form equation of each ellipse.

7. Foci: $(\sqrt{17}, 0), (-\sqrt{17}, 0)$ +c −c a=9 c=$\sqrt{17}$
Endpoints of major axis: (9, 0), (−9, 0) a a

$\frac{x^2}{81}+\frac{y^2}{64}=1$

$a^2=b^2+c^2$
$81=b^2+17$
$b^2=64$ b=8

$(\sqrt{17})^2=9^2-b^2$
$17=81-b^2$
$64=b^2$
$8=b$

8. Foci: $(\sqrt{115}, 0), (-\sqrt{115}, 0)$ a=$\sqrt{195}$ c=$\sqrt{115}$
Endpoints of major axis: $(\sqrt{195}, 0), (-\sqrt{195}, 0)$

$\frac{x^2}{195}+\frac{y^2}{80}=1$

$115=195+b^2$
$80=b^2$
$\sqrt{80}=b$

9. Center: (6, −5) 12=a
Vertex: (6, 7)
Focus: $(6, -5-6\sqrt{3})$ (h, k−c)
vertical

$\frac{(x-6)^2}{36}+\frac{(y+5)^2}{144}=1$

a=12
b=
c=$6\sqrt{3}$

$108=144-b^2$
$b^2=144-108=36$

10. Center: (7, −10) h k
Vertex: (−6, −10) a k
Co-vertex: (7, −17) h b
horizontal

$\frac{(x-7)^2}{169}+\frac{(y+10)^2}{49}=1$

a=13
b=7
c=NA

11. Center: (1, −7)
Vertex: (1, 1) a=8
$c^2=55$
vertical

$55=64-b^2$

$\frac{(x-1)^2}{9}+\frac{(y+7)^2}{64}=1$ b=3

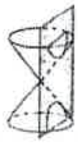
12. Center: (4, 0)
Focus: $(4, 3\sqrt{7})$
Width: 18 → minor axis
vertical

$\frac{(x-4)^2}{81}+\frac{y^2}{144}=1$

a=
b=9
c=$3\sqrt{7}$

$(3\sqrt{7})^2=a^2-81$
$63=a^2-81$
$144=a^2$
$12=a$

Hyperbolas



Hyperbola: The set of all points P in a plane such that the absolute value of the difference between the distances from P to two fixed points F_1 & F_2 is constant.

The standard form of the equation of a parabola with vertex (h,k)

$$\boxed{\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1}$$

Focus of the hyperbola: The two fixed points on traverse axis

Vertex: the turning point of each branch of the hyperbola.
"a" value

Transverse axis: the segment connecting the two vertices

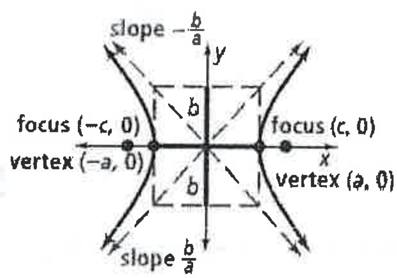
Axis of symmetry: the axis on which the traverse axis lies.

Center of the hyperbola: midpoint between the two vertices.

*"a" is under the positive term (Not the BIGGEST)

Key Concept Properties of Hyperbolas with Center (0, 0)

Horizontal Hyperbola



Equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

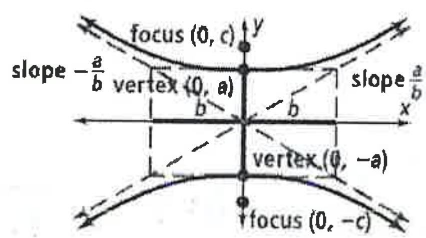
Transverse axis: Horizontal

Vertices: $(\pm a, 0)$

Foci: $(\pm c, 0)$, where $c^2 = a^2 + b^2$ ☆

Asymptotes: $y = \pm\frac{b}{a}x$

Vertical Hyperbola



Equation: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Transverse axis: Vertical

Vertices: $(0, \pm a)$

Foci: $(0, \pm c)$, where $c^2 = a^2 + b^2$

Asymptotes: $y = \pm\frac{a}{b}x$

Key Concept *Equations of Hyperbolas with Centers at (h, k)*

Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Direction of Transverse Axis	horizontal	vertical
Equations of Asymptotes	$y - k = \pm\frac{b}{a}(x - h)$	$y - k = \pm\frac{a}{b}(x - h)$

Vertices: $(x \pm a, y)$ $(x, y \pm a)$
*Foci $\pm c$ $\pm c$

x first: opens L/R

y first: opens up/down

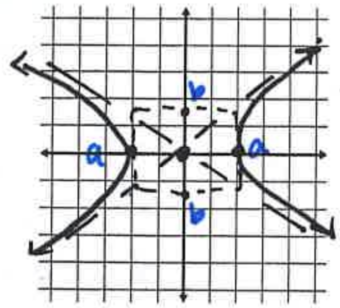
Example 15: A hyperbola centered at (0,0) has vertices ($\overset{a}{\pm 4}$,0) and one focus ($\overset{c}{5}$,0) write the standard form equation of the hyperbola and graph it.

$c^2 = a^2 + b^2$
$b^2 = c^2 - a^2$
$b^2 = 25 - 16$
$b^2 = 9$
$b = 3$

$$\frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

asymptotes: $y = \pm\frac{3}{4}x$



Example 16: What are the vertices, foci and asymptotes of the hyperbola with equation $9y^2 - 7x^2 = 63$
Graph the hyperbola.

$$\frac{9y^2}{63} - \frac{7x^2}{63} = \frac{63}{63}$$

2.6 → $a = \sqrt{7}$ Vertices: $(0, \pm\sqrt{7})$

$b = 3$ co: $(\pm 3, 0)$

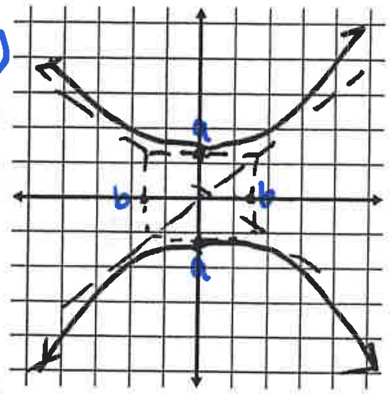
$$\frac{9y^2}{63} - \frac{7x^2}{63} = 1$$

$$\frac{y^2}{7} - \frac{x^2}{9} = 1$$

$c^2 = 9 + 7$
$c^2 = 16$
$c = 4$

Foci: $(0, \pm 4)$

Asymptotes: $y = \pm\frac{3}{\sqrt{7}}x$



Identify the vertices, foci, and direction of opening of each.

1. $\frac{y^2}{25} - \frac{x^2}{16} = 1$

$a = 5$ $c^2 = 25 + 16$
$b = 4$ $c^2 = 41$
$c = \sqrt{41}$

vertices $(0, \pm 5)$
foci $(0, \pm\sqrt{41})$
opens up/ down

2. $\frac{x^2}{121} - \frac{y^2}{36} = 1$

$a = 11$
$b = 6$
$c^2 = 121 + 36$
$c^2 = 157$
$c = \sqrt{157}$

Vertices $(\pm 11, 0)$
foci $(\pm\sqrt{157}, 0)$
opens L-R

3. $\frac{(x+2)^2}{169} - \frac{(y+8)^2}{4} = 1$ $c = (-2, -8)$

$a = 13$ $b = 2$

Vertices: $(11, -8)$
$(-15, -8)$
foci: $(-2 + \sqrt{173}, -8)$
$(-2 - \sqrt{173}, -8)$
opens L-R

$c^2 = 169 + 4$
$c^2 = 173$
$c = \sqrt{173}$

$(-2+13, -8)$ $(-2-13, -8)$

4. $\frac{(y+8)^2}{36} - \frac{(x+2)^2}{25} = 1$ $C: (-2, -8)$

$a = 6$ $b = 5$

$(-2, -8+6)$
$(-2, -8-6)$

Vertices: $(-2, -2)$ $(-2, -14)$
Foci: $(-2, -8+\sqrt{61})$, $(-2, -8-\sqrt{61})$
Opens up/down

Don't stress about graphing!

Identify the vertices and foci of each. Then sketch the graph. Horiz

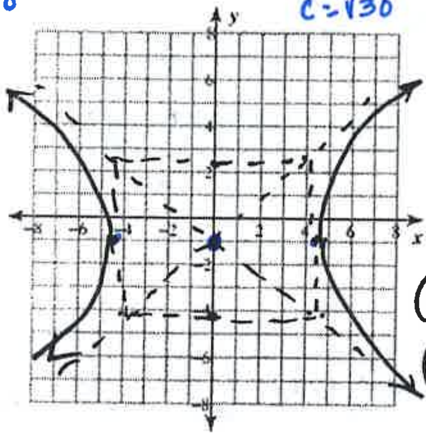
Horiz.

5. $\frac{x^2}{20} - \frac{(y+1)^2}{10} = 1$

$a = \sqrt{20}$
$b = \sqrt{10}$
$c = \sqrt{30}$

$C: (0, -1)$

$c^2 = 20 + 10$
$c^2 = 30$



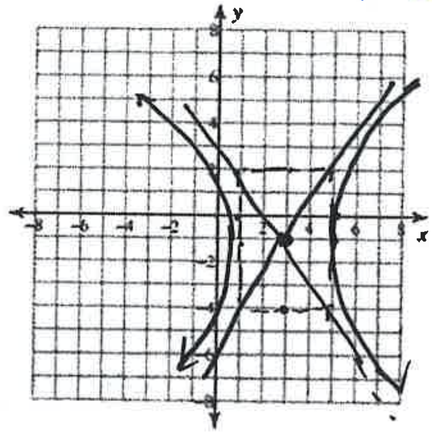
Vertices
$(2\sqrt{5}, -1)$
$(-2\sqrt{5}, -1)$

Foci
$(\sqrt{30}, -1)$
$(-\sqrt{30}, -1)$

6. $\frac{(x-3)^2}{4} - \frac{(y+1)^2}{9} = 1$

$C: (3, -1)$

$a = 2$ $b = 3$ $c = \sqrt{13}$



Vertices: $(5, -1)$
$(1, -1)$

Foci $(3 + \sqrt{13}, -1)$
$(3 - \sqrt{13}, -1)$

$c^2 = 4 + 9$
$c^2 = 13$
$c = \sqrt{13}$

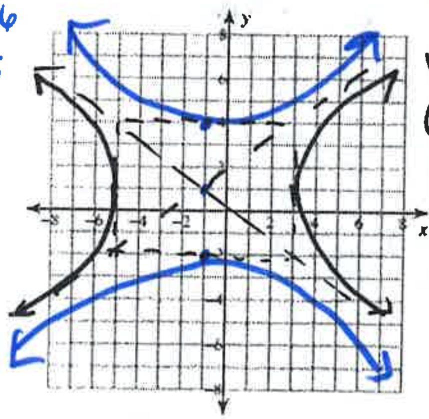
Vert.

7. $\frac{(y-1)^2}{9} - \frac{(x+1)^2}{16} = 1$

$a = 3$
$b = 4$
$c = 5$

$C: (-1, 1)$

$c^2 = 9 + 16$
$c^2 = 25$
$c = 5$



Vertices:
$(-1, 4)$ $(-1, 1+3)$
$(-1, -2)$ $(-1, 1-3)$

foci $(-1, 6)$
$(-1, 4)$

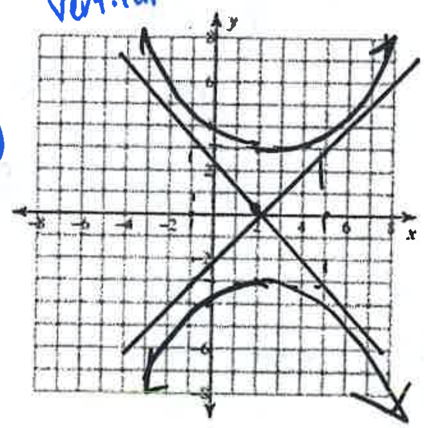
8. $\frac{y^2}{9} - \frac{(x-2)^2}{9} = 1$

vertical

$a = 3$
$b = 3$

$c = \sqrt{18}$

$C: (2, 0)$



Vertices: $(2, 3)$
$(2, -3)$

foci $(2, 3\sqrt{2})$
$(2, -3\sqrt{2})$

$c^2 = 9 + 9$
$c^2 = 18$

Use the information provided to write the standard form equation of each hyperbola.

9. $-x^2 + y^2 - 18x - 14y - 132 = 0$

$\frac{(y-7)^2}{100} - \frac{(x+9)^2}{100} = 1$

10. $9x^2 - 4y^2 - 90x + 32y - 163 = 0$

$\frac{(x-5)^2}{36} - \frac{(y-4)^2}{81} = 1$

11. Vertices: $(8, 14)$, $(8, -10)$
Conjugate Axis is 6 units long

$\frac{(y-2)^2}{144} - \frac{(x-8)^2}{9} = 1$

12. Vertices: $(4, 9 + \sqrt{30})$, $(4, 9 - \sqrt{30})$
Conjugate Axis is $2\sqrt{195}$ units long

$\frac{(y-9)^2}{30} - \frac{(x-4)^2}{195} = 1$

Precalculus

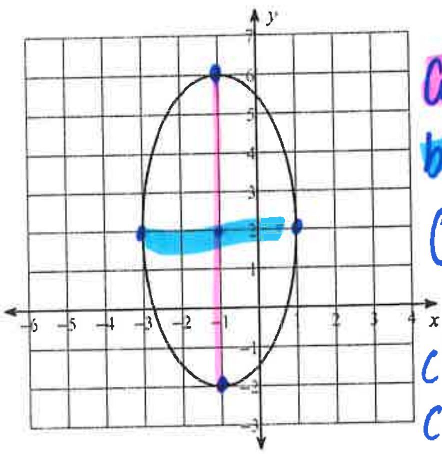
Name________________________ ID: 1

Conic Sections Review

Date________________ Period____

Given the graph, identify the center, the foci and the vertices. Then write the equation for the ellipse

1)

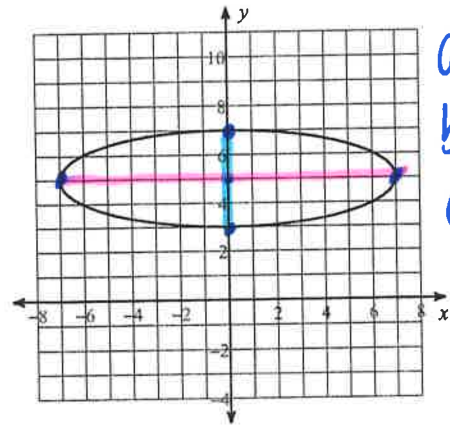


a=4
b=2
$c=\sqrt{12}$
$c^2=16-4$
$c^2=12$

Eq: $\frac{(x+1)^2}{4}+\frac{(y-2)^2}{16}=1$

Center: (-1, 2)
Vertices: (-1, -2) (-1, 6)
foci: $(-1, 2+\sqrt{12})$ $(-1, 2-\sqrt{12})$

2)

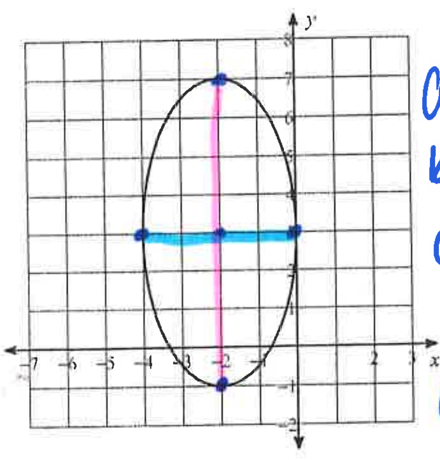


a=7
b=2
$c=\sqrt{45}$
$c^2=49-4$
$c^2=45$

Eq: $\frac{x^2}{49}+\frac{(y-5)^2}{4}=1$

Center: (0, 5)
Vertices: (-7, 5) (7, 5)
foci: $(\sqrt{45}, 5)$ $(-\sqrt{45}, 5)$

3)

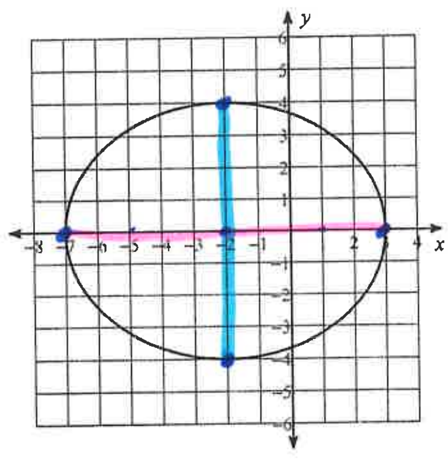


a=4
b=2
$c=\sqrt{12}$
$c^2=16-4$
$c^2=12$

Eq: $\frac{(x+2)^2}{4}+\frac{(y-3)^2}{16}=1$

Center: (-2, 3)
Vertices: (-2, 7) (-2, -1)
foci: $(-2, 3+\sqrt{12})$ $(-2, 3-\sqrt{12})$

4)



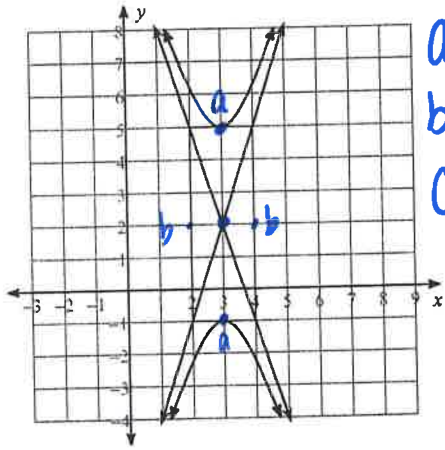
a=5
b=4
c=3
$c^2=25-16$
$c^2=9$
c=

Eq: $\frac{(x+2)^2}{25}+\frac{y^2}{16}=1$

Center: (-2, 0)
Vertices: (-7, 0) (3, 0)
foci: (1, 0) (-5, 0)
-2+3 -2-3

Given the graph, identify the center, the vertices and the foci. Then write the equation of the hyperbola.

5)



$a = 3$

$b = 1$

$c = \sqrt{10}$

$c^2 = 9 + 1$

$c^2 = 10$

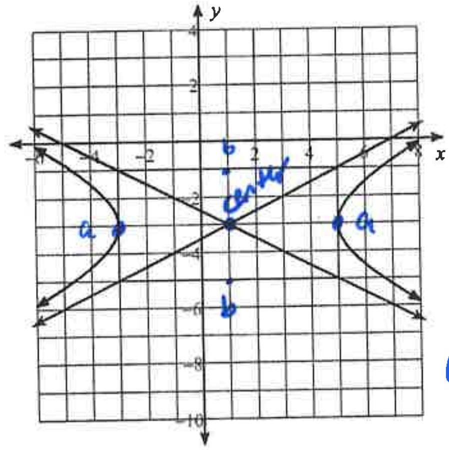
Eq: $\frac{(y-2)^2}{9} - (x-3)^2 = 1$

Center: $(3, 2)$

Vertices: $(3, -1)$
$(3, 5)$

Foci: $(3, 2 \pm \sqrt{10})$

6)



$a = 4$

$b = 2$

$c = \sqrt{20}$

$c^2 = 16 + 4$

$c^2 = 20$

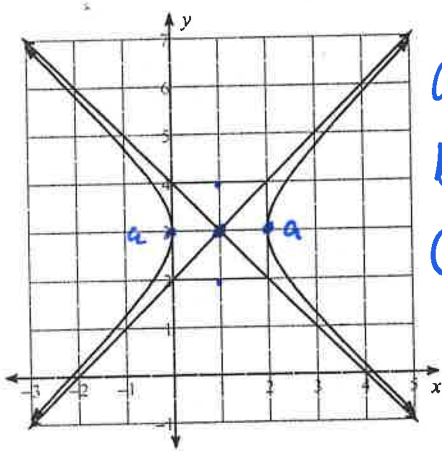
Eq: $\frac{(x-1)^2}{16} - \frac{(y+3)^2}{4} = 1$

Center: $(1, -3)$

Vertices: $(5, 3)$
$(-3, 3)$

Foci: $(1 \pm \sqrt{20}, 3)$

7)



$a = 1$

$b = 1$

$c = \sqrt{2}$

$c^2 = 1 + 1$

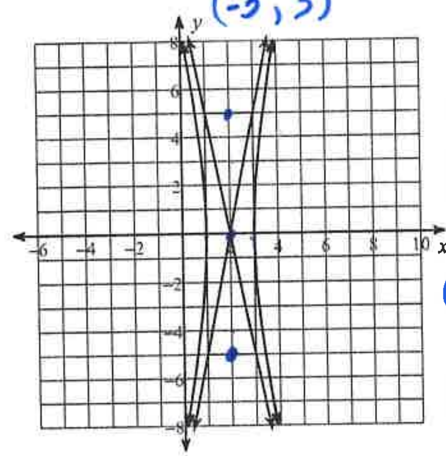
Eq: $(x-1)^2 - (y-3)^2 = 1$

Center: $(1, 3)$

Vertices: $(2, 3)\ (0, 3)$

Foci: $(1 \pm \sqrt{2}, 3)$

8)



$a = 1$

$b = 5$

$c = \sqrt{26}$

$c^2 = 1 + 25$

$c^2 = 26$

Eq: $(x-2)^2 - \frac{y^2}{25} = 1$

Center: $(2, 0)$

Vertices: $(3, 0)\ (1, 0)$

Foci: $(2 \pm \sqrt{26}, 0)$

Use the information provided to write the standard form equation of each ellipse.

9) $4x^2 + 81y^2 + 32x + 324y + 64 = 0$

$$\frac{(x+4)^2}{81} + \frac{(y+2)^2}{4} = 1$$

$4x^2 + 32x + 81y^2 + 324y = -64$

$4(x^2 + 8x + 16) + 81(y^2 + 4y + 4) = -64 + 64 + 324$

$\frac{4(x+4)^2}{324} + \frac{81(y+2)^2}{324} = \frac{324}{324}$

$\frac{(x+4)^2}{81} + \frac{(y+2)^2}{4} = 1$

10) $x^2 + 4y^2 + 16x - 36 = 0$

$$\frac{(x+8)^2}{100} + \frac{y^2}{25} = 1$$

$x^2 + 16x + 4y^2 = 36$

$(x^2 + 16x + 64) + 4y^2 = 36 + 64$

$\frac{(x+8)^2}{100} + \frac{4y^2}{100} = \frac{100}{100}$

$\frac{(x+8)^2}{100} + \frac{y^2}{25} = 1$

Use the information provided to write the standard form equation of each hyperbola. Positive FIRST

11) $-4x^2 + y^2 + 56x + 12y - 176 = 0$

$$\frac{(y+6)^2}{16} - \frac{(x-7)^2}{4} = 1$$

$-4x^2 + 56x + y^2 + 12y = 176$

$-4(x^2 - 14x + 49) + (y^2 + 12y + \quad) = 176 +$

$(y^2 + 12y + 36) - 4(x^2 - 14x + 49) = 176 + 36 + (-196)$

$\frac{(y+6)^2}{16} - \frac{4(x-7)^2}{16} = \frac{16}{16}$

$\frac{(y+6)^2}{16} - \frac{(x-7)^2}{4} = 1$

12) $x^2 - y^2 - 18x - 6y + 56 = 0$

$$\frac{(x-9)^2}{16} - \frac{(y+3)^2}{16} = 1$$

$x^2 - 18x - y^2 - 6y = -56$

$(x^2 - 18x + 81) - (y^2 + 6y + 9) = -56 + 81 - 9$

$(x-9)^2 - (y+3)^2 = 16$

$\frac{(x-9)^2}{16} - \frac{(y+3)^2}{16} = 1$

Use the information provided to write the standard form equation of each ellipse. $(h, k \pm c)$

13) Endpoints of major axis: $(16, -7)$, $(-6, -7)$ Horiz.
Endpoints of minor axis: $(5, -1)$, $(5, -13)$

22

12

Eq: $\dfrac{(x-5)^2}{121} + \dfrac{(y+7)^2}{36} = 1$

$a = 11$ $b = 6$

Center: $(5, -7)$

$16 - 11_a$ OR $-6 + 11_a$

$-1 - 6_b$ OR $-13 + 6_b$

14) Foci: $(6, 8)$, $(6, -16)$ vertical
Co-vertices: $(11, -4)$, $(1, -4)$ Endpts. of minor axis

24

10

Eq: $\dfrac{(x-6)^2}{25} + \dfrac{(y+4)^2}{169} = 1$

$a = 13$ $b = 5$ $c = 12$

Center: $(6, -4)$

Same as Foci

Same as co-vert.

$12^2 = a^2 - 5^2$

$144 = a^2 - 25$

$169 = a^2$

$13 = a$

Use the information provided to write the standard form equation of each hyperbola.

15) Vertices: $(0, 22)$, $(0, -2)$ $24 \Rightarrow a = 12$
Foci: $(0, 10 + \sqrt{265})$, $(0, 10 - \sqrt{265})$

"k" ± "c"

Eq: $\dfrac{(y-10)^2}{144} - \dfrac{x^2}{121} = 1$

$a = 12$ $b = 11$ $c = \sqrt{265}$

Center: $(0, 10)$

Same as vertices

$(\sqrt{265})^2 = 12^2 + b^2$

$265 = 144 + b^2$

$121 = b^2$

$11 = b$

* y changed, so it's vertical

16) Vertices: $(13, 2)$, $(-11, 2)$ $24 \Rightarrow a = 12$
Foci: $(1 + \sqrt{265}, 2)$, $(1 - \sqrt{265}, 2)$ $(h \pm c, k)$

Eq: $\dfrac{(x-1)^2}{144} - \dfrac{(y-2)^2}{121} = 1$

$a = 12$ $b = 11$ $c = \sqrt{265}$

Center: $(1, 2)$

foci

same as vertex + foci

* x-changed, so it's horizontal

$c^2 = a^2 + b^2$

$(\sqrt{265})^2 = 12^2 + b^2$

$\vdots$

$11 = b$