Composite Functions

Students will be able to form composite functions and decompose functions.

Honors Precalculus/ Precalculus

Composite Functions

A composite function is the result of substituting one function for the variable of another function.

Example 1: find the composition of the function f with g and the composition of the function g with f. Then the composition of j with k and the composition of k with j.

$$f(x) = x + 5 \qquad g(x) = 10x^2 + 3x + 1$$

a) $(f \circ g) = f(g(x)) = (10x^2 + 3x + 1) + 5$

$= 10x^2 + 3x + 6$

b) $(g \circ f) = g(f(x)) = 10(x+5)^2 + 3(x+5) + 1$

$= 10(x^2 + 10x + 25) + 3x + 15 + 1$

$= 10x^2 + 100x + 250 + 3x + 15 + 1$

$= 10x^2 + 103x + 266$

$$j(x) = \frac{5x}{x} \qquad k(x) = \frac{10}{6x}$$

c. $(j \circ k) = j(k(x)) = \dfrac{\frac{5}{1}\left(\frac{10}{6x}\right)}{\frac{10}{6x}} = \dfrac{\frac{50}{6x}}{\frac{10}{6x}} = \dfrac{50}{6x} \cdot \dfrac{6x}{10} = 5$

d. $(k \circ j) = k(j(x)) = \dfrac{10}{\frac{6}{1}\left(\frac{5x}{x}\right)} = \dfrac{10}{\frac{30x}{x}} = \dfrac{10}{30} = \dfrac{1}{3}$

Example 2: find the composition of the function ***f*** with ***g*** and the composition of the function ***g*** with ***f***.

$f(x) = 4x$ $\qquad g(x) = \frac{3}{x+2}$

a) $(f \circ g) = f(g(x))$ $= \frac{4}{1}\left(\frac{3}{x+2}\right) = \frac{12}{x+2}$

b) $(g \circ f) = g(f(x))$ $= \frac{3}{4x+2}$

c) $g(f(1))$ $= \frac{3}{4(1)+2} = \frac{3}{6} = \frac{1}{2}$

d) $f(g(4))$ $= \frac{12}{4+2} = \frac{12}{6} = 2$

Example 3:

A consumer advocacy company conducted a study to research the pricing of fruits and vegetables. They collected data on the size and price of produce items, including navel oranges. They found that, for a given chain of stores, the price of oranges was a function of the weight of the oranges, $p = f(w)$.

w weight in pounds	0.2	0.25	0.3	0.4	0.5	0.6	0.7
p price in dollars	0.26	0.32	0.39	0.52	0.65	0.78	0.91

The company also determined that the weight of the oranges measured was a function of the radius of the oranges, $w = g(r)$.

r radius in inches	1.5	1.65	1.7	1.9	2	2.1
w weight in pounds	0.38	0.42	0.43	0.48	0.5	0.53

Use the table to evaluate $f(g(2))$ $= 0.65$

Decomposing Functions

Decomposing a function means reversing a composite. Identify ***f(x)*** and ***g(x)***. Think of ***g(x)*** as the inside function and ***f(x)*** is the outside function.

$f(g(x))$

Example 4: Decompose the function.

a) $h(x) = (3x - 5)^2$

$g(x) = 3x-5$
$f(x) = x^2$

b) $h(x) = \frac{5}{\sqrt{x+12}}$

$g(x) = \sqrt{x+12}$
$f(x) = \frac{5}{x}$

c) $h(x) = \sqrt[3]{x^2 - 8}$

$g(x) = x^2-8$
$f(x) = \sqrt[3]{x}$

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Composition of Functions Day 1

Find the indicated composition of the value of the composition. Let $f(x) = 2x - 1$, $g(x) = 3x$ and $h(x) = x^2 - 1$.

1. $(f \circ g) = 2(3x) - 1$
$= 6x - 1$

2. $(g \circ h) = 3(x^2 - 1)$
$= 3x^2 - 3$

3. $(g \circ f) = 3(2x - 1)$
$= 6x - 3$

4. $(h \circ f) = (2x-1)^2 - 1$
$= 4x^2 - 4x + 1 - 1$
$= 4x^2 - 4x$

5. $(h \circ g) = (3x)^2 - 1$
$= 9x^2 - 1$

6. $(g \circ h)(24) = 3(24)^2 - 3$
$= 1728 - 3$
$= 1725$

7. $(f(h(7))$
$(f(h)) = 2(x^2-1) - 1$
$= 2x^2 - 3$
$f(h(7)) = 2(7)^2 - 3$
$= 98 - 3 = 95$

8. $(g(f(9))) = 6(9) - 3$
$= 51$

9. $(h \circ f)(2a)$
$4(2a)^2 - 4(2a)$
$16a^2 - 8a$

10. $h(g(24)) =$
$9(24)^2 - 1$
$5184 - 1 = 5183$

11. A soda factory makes it is own carbonated water. The cost of a bottle of soda depends on the yield of carbon dioxide to water. The yield will vary with temperature.

The cost to manufacture the soda is $c = b(g)$.

g yield of CO_2 in cubic milliliters	10	20	30	40	50	60
c Soda Cost per Bottle in dollars	.012	.010	.008	.006	.004	.002

The yield of carbon dioxide is a function of temperature $g = f(t)$.

t Temperature in °F	32°	44°	56°	68°	80°	92°
g yield of CO_2 in cubic milliliters	10	20	30	40	50	60

Use the tables to evaluate:

a. $b(f(68°)$ = .006

b. $b(f(56°)$ = .008

c. If the plant is located in a region where the average temperature is 65° what is the average cost to manufacture a bottle of soda?

$\left(\frac{.002}{12}\right) \times 9 = .0015$

avg cot = .008 - .0015 = \$.0065

12. Find $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$.

a. $h(x) = (x+7)^2$

$g(x) = x+7$
$f(x) = x^2$

b. $h(x) = \frac{\sqrt[3]{(x-1)}}{(x-1)}$

$g(x) = x-1$
$f(x) = \frac{\sqrt[3]{x}}{x}$

Honors Precaclulus/Precalculus Name: Key Date: ______ Period: ____

Composition of Functions Day 2

Find $f \circ g$ and $g \circ f$, and evaluate the composition for given value.

1. $f(x) = 2x - 3;\ \ g(x) = x + 1\ ;\ (f \circ g)(3) and (g \circ f)(-2).$

$f(g(x)) = 2(x+1) - 3 = 2x + 2 - 3 = 2x - 1$ $\quad f(g(3)) = 2(3) - 1 = 5$

$g(f(x)) = 2x - 3 + 1 = 2x - 2$ $\quad g(f(-2)) = 2(-2) - 2 = -6$

2. $f(x) = x^2 + 4;\ \ g(x) = \sqrt{x+1}\ ;\ (f \circ g)(-3) and (g \circ f)(2).$

$f(g(x)) = (\sqrt{x+1})^2 + 4 = x + 1 + 4 = x + 5$ $\quad f(g(-3)) = -3 + 5 = 2$

$g(f(x)) = \sqrt{x^2+4+1} = \sqrt{x^2+5}$ $\quad g(f(2)) = \sqrt{(2)^2+5} = \sqrt{9} = \pm 3$

3. $f(x) = x^2 - 1;\ \ g(x) = \frac{1}{x-1}\ ;\ (f \circ g)(-1) and (g \circ f)(4).$

$f(g(x)) = \left(\frac{1}{x-1}\right)^2 - 1 = \frac{1}{x^2-2x+1} - 1 = 1 - x^2 + 2x - 1 = -x^2 + 2x =$

$f(g(-1)) = -(-1)^2 + 2(-1) = -1 - 2 = -3$

$g(f(x)) = \frac{1}{x^2-1-1} = \frac{1}{x^2-2}$ $\quad g(f(4)) = \frac{1}{(4)^2-2} = \frac{1}{14}$

4. $f(x) = \frac{1}{2x};\ \ g(x) = \frac{1}{3x}\ ;\ (f \circ g)(0) and (g \circ f)\left(-\frac{3}{2}\right).$

$f(g(x)) = \frac{1}{\frac{2}{1}\left(\frac{1}{3x}\right)} = \frac{1}{\frac{2}{3x}} = \frac{3x}{2}$ $\quad f(g(0)) = \frac{3(0)}{2} = 0$

$g(f(x)) = \frac{1}{\frac{3}{1}\left(\frac{1}{2x}\right)} = \frac{1}{\frac{3}{2x}} = \frac{2x}{3}$ $\quad g(f(-\frac{3}{2})) = \frac{2}{3} \cdot \frac{-3}{2} = -1$

5. $f(x) = x^3;\ \ g(x) = \sqrt[3]{1 - x^3}\ ;\ (f \circ g)(0) and (g \circ f)(1).$

$f(g(x)) = \left(\sqrt[3]{1-x^3}\right)^3 = 1 - x^3$ $\quad f(g(0)) = 1 - 0^3 = 1$

$g(f(x)) = \sqrt[3]{1-(x^3)^3} = \sqrt[3]{1-x^9}$ $\quad g(f(0)) = \sqrt[3]{1-(1)^9} = \sqrt[3]{0} = 0$

Application Problems

6. A commercial bakery makes a mango meltaway cookie. The cost to make the cookie depends on the diameter of the mango pit. The size of mango pit depends the average temperature during the growing season.

The cost to manufacture the cookie is ***c = f(m)***

m Size of Mango Pit in millimeters	20	30	40	50	60	70
c Cookie Cost per cookie in dollars	.005	.008	.012	0.17	.023	.030

The size of the pit is a function of temperature ***m= g(t).***

t Temperature in °F	80°	85°	90°	95°	100°	105°
m Size of Mango Pit in millimeters	18	24	32	40	50	62

Use the tables to evaluate *f(g(95°)* = 0.012

7. The size of a baby tarantula depends upon the number of eggs laid by the mother. The number of eggs laid by the mother depends upon the age of the mother. ***s = f(n)***

n number of eggs	100	120	140	160	180	200
s size of tarantula (in.)	10	9	8	7	.6	5

The size of the pit is a function of temperature ***n= g(a).***

a Age in years	4	8	12	16	20	24
n number of eggs	100	120	140	160	180	200

$f(g(8)) = 9$

~~Use the tables to evaluate f(g(8)~~

8. The number of cherry tomatoes produced by a single plant depends upon the amount of acid found in the soil. The amount of acid in the soil depends upon the zone that soils is located in. ***t = f(a)***

a % acid	10	15	20	25	30	35
t number of tomatoes	240	200	160	120	80	40

The size of the pit is a function of temperature ***a= g(z).***

z zone	1	2	3	4	5	6
a acid	35	30	25	20	15	10

Use the tables to evaluate *f(g(5)* = 200

9. Find *f*(*x*) and g(*x*) so that *h*(*x*) = *f*(g(*x*)). (There may be more than one answer.) DECOMPOSE

a. $h(x) = \sqrt{x-7}$

g(x) = $x-7$

f(x) = $\sqrt{x}$

b. $h(x) = (x^3-3)^2$

g(x) = x^3-3

f(x) = x^2

c. $h(x) = \frac{2}{\sqrt{3x+5}}$

g(x) = $\sqrt{3x+5}$

f(x) = $\frac{2}{x}$

d. $h(x) = \left|(\sqrt{x}-3)^2\right|$

g(x) = $(\sqrt{x}-3)^2$

f(x) = $|x|$

10. Find g(*f*(*x*)) and *f*(g(*x*)) for $f(x) = \frac{3}{4x}$ *and* $g(x) = \frac{x}{2x}$

$$g(f(x)) = \frac{\frac{3}{4x}}{\frac{2}{1}\left(\frac{3}{4x}\right)} = \frac{\frac{3}{4x}}{\frac{6}{4x}} = \frac{3}{4x} \cdot \frac{4x}{6} = \frac{3}{6} = \frac{1}{2}$$

$$f(g(x)) = \frac{3}{\frac{4}{1}\left(\frac{x}{2x}\right)} = \frac{3}{\frac{4x}{2x}} = \frac{3}{2}$$