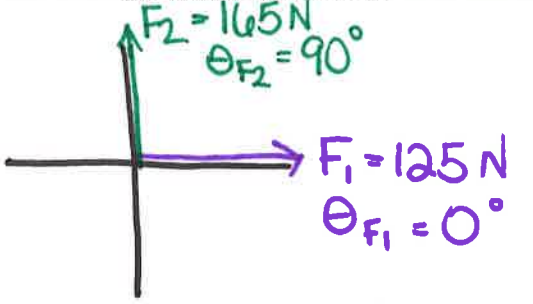
Vector Practice Problems

Name Key

Draw vector diagrams to solve each problem.

1) Two boys push on a box. One pushes with a force of 125 N to the east. The other exerts a force of 165 N to the north. What is the size and direction of the resultant force on the box?



$\vec{F_1} = \langle 125\cos 0, 125\sin 0° \rangle = \langle 125, 0 \rangle$

$\vec{F_2} = \langle 165\cos 90, 165\sin 90° \rangle = \langle 0, 165 \rangle$

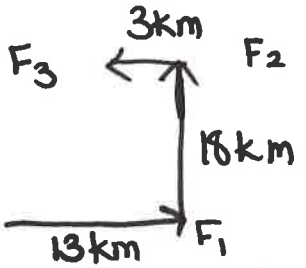
$\vec{F_1} + \vec{F_2} = \langle 125, 165 \rangle$

$|F_1 + F_2| = \sqrt{125^2 + 165^2} \approx 207$ N

$\theta_{F_1+F_2} = \cos^{-1}\left(\frac{125}{207}\right) \approx 52.85°$

2) An explorer walks 13 km due east, then 18 km north, and finally 3 km west.
   a) What is the <u>total distance</u> walked?
   b) What is the <u>displacement</u> of the explorer (current distance from the starting point)?

a) Total Dist. = 13 + 18 + 3 = 34 km



b) $\vec{F_1} = \langle 13\cos 0°, 13\sin 0° \rangle = \langle 13, 0 \rangle$

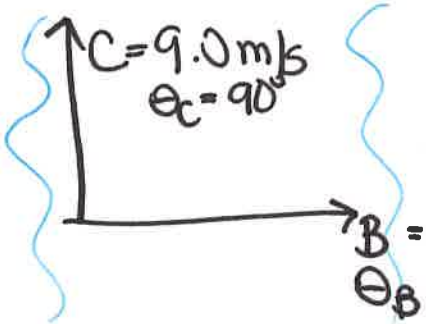
$\vec{F_2} = \langle 18\cos 90°, 18\sin 90° \rangle = \langle 0, 18 \rangle$

$\vec{F_3} = \langle 3\cos 180, 3\sin 180° \rangle = \langle -3, 0 \rangle$

$F_1 + F_2 + F_3 = \langle 10, 18 \rangle$

displacement $= \sqrt{10^2 + 18^2} = \sqrt{100 + 324} = \sqrt{424} \approx 20.59$ km

3) A motorboat heads due east at 16 m/s across a river that flows due north at 9.0 m/s.
   a) What is the resultant velocity (<u>speed</u> and <u>direction</u>) of the boat?
   b) If the river is 136 m wide, how long does it take the motorboat to reach the other side?
   c) How far downstream is the boat when it reaches the other side of the river?



a) $\vec{C} = \langle 9\cos 90°, 9\sin 90° \rangle = \langle 0, 9 \rangle$

$\vec{B} = \langle 16\cos 0°, 16\sin 0° \rangle = \langle 16, 0 \rangle$

$\vec{C} + \vec{B} = \langle 16, 9 \rangle$

$\|\vec{C} + \vec{B}\| = \sqrt{16^2 + 9^2} = 18.36$ m/s

$\theta_{C+B} = \cos^{-1}\left(\frac{16}{18.36}\right) \approx 29.36°$

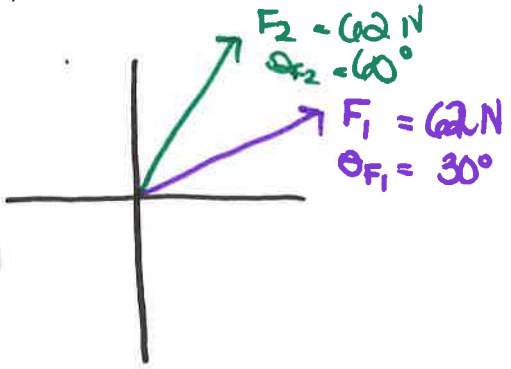
b) $d = rt = 136 = \frac{16\text{ m}}{\text{s}} \cdot t \quad t = \frac{136}{16} \approx 8.5$ s

c) $d = rt$

$d = \frac{9\text{ m}}{\text{s}} \cdot \frac{8.5\text{ s}}{1}$

$d = 76.5$ m

4) A 62-N force acts on an object at 30° and a second 62-N force acts at 60°. Determine the resultant force.



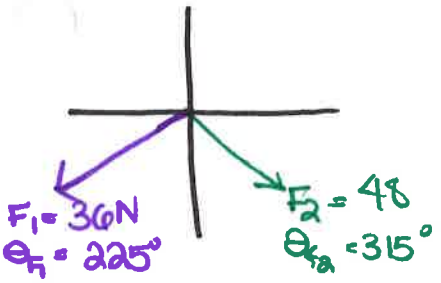
$\vec{F_1} = \langle 62\cos 30°, 62\sin 30° \rangle = \langle 53.694, 31 \rangle$

$\vec{F_2} = \langle 62\cos 60, 62\sin 60 \rangle = \langle 31, 53.694 \rangle$

$\vec{F_1} + \vec{F_2} = \langle 84.964, 84.694 \rangle$

$|\vec{F_1} + \vec{F_2}| = \sqrt{84.694^2 + 84.694^2} \approx 119.775$ N

5) Two forces act on an object. A 36-N force acts at 225°. A 48-N force acts at 315°.
What would be the magnitude and direction of the resultant force?



$F_1 = \langle 36\cos 225°, 36\sin 225° \rangle = \langle -25.456, -25.456 \rangle$

$F_2 = \langle 48\cos 315, 48\sin 315 \rangle = \langle 33.941, -33.941 \rangle$

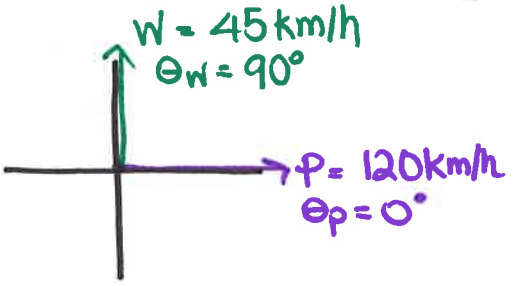
$= \langle 8.485, -59.397 \rangle$

$F_1 + F_2$

$|F_1 + F_2| = \sqrt{(8.485)^2 + (-59.397)^2} \approx 60\text{N}$

$\theta_{F_1+F_2} = \cos^{-1}\left(\frac{8.485}{60}\right) \approx 81.87°$ $\quad \theta_{ACTUAL} = 360 - 81.87 = 278.13°$

6) While flying due east at 120 km/h, an airplane is also carried northward at 45 km/h by the wind blowing due north. What is the plane's resultant velocity?



$\vec{W} = \langle 45\cos 90°, 45\sin 90° \rangle = \langle 0, 45 \rangle$

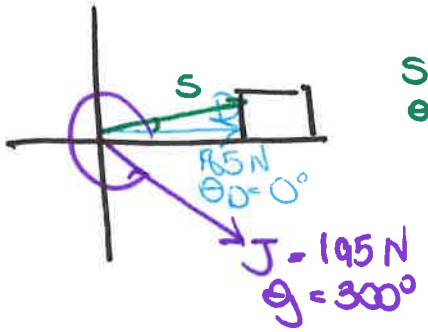
$\vec{P} = \langle 120\cos 0°, 120\sin 0° \rangle = \langle 120, 0 \rangle$

$= \langle 120, 45 \rangle$

$\vec{W} + \vec{P}$

$|\vec{W} + \vec{P}| = \sqrt{120^2 + 45^2} \approx 128.16 \text{ km/h}$

7) Three teenagers push a heavy crate across the floor. Dion pushes with a force of 185 N at 0°.
Shirley exerts a force of 165 N at 30°, while Joan pushes with 195 N force at 300°.
What is the resultant force on the crate? What direction (angle) does the crate go?



$\vec{D} = \langle 185\cos 0, 185\sin 0° \rangle = \langle 185, 0 \rangle$

$\vec{S} = \langle 165\cos 30°, 165\sin 30° \rangle = \langle 142.899, 82.5 \rangle$

$\vec{J} = \langle 195\cos 300, 195\sin 300° \rangle = \langle 97.5, -168.874 \rangle$

$\vec{D} + \vec{S} + \vec{J}$ $\quad = \langle 425.394, -86.375 \rangle$

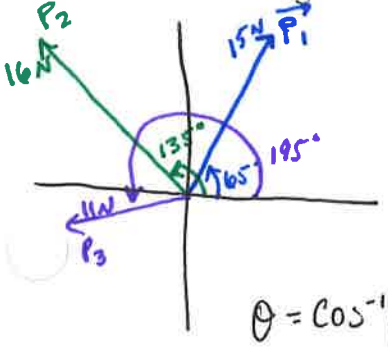
$|D+S+J| = \sqrt{425.394^2 + (-86.375)^2}$

≈ 434.075

$\theta_{D+S+J} = \cos^{-1}\left(\frac{425.394}{434.075}\right) \approx 11.476$

$\theta = 360 - 11.476 = 348.522°$

8) Three people are pulling on a tree.
The first person pulls with 15 N at 65°; the second with 16 N at 135°; the third with 11 N at 195°.
What is the magnitude and direction of the resultant force on the tree?



$\vec{P_1} = \langle 15\cos 65, 15\sin 65 \rangle = \langle 6.339, 13.595 \rangle$

$\vec{P_2} = \langle 16\cos 135, 16\sin 135 \rangle = \langle -11.314, 11.314 \rangle$

$\vec{P_3} = \langle 11\cos 195, 11\sin 195 \rangle = \langle -10.625, -2.847 \rangle$

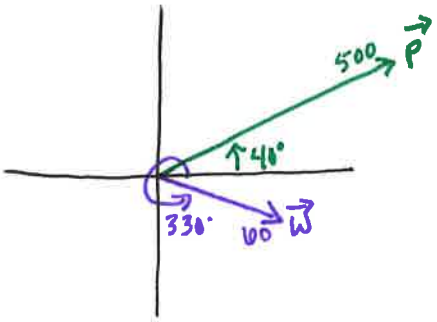
$\vec{P_1} + \vec{P_2} + \vec{P_3} = \langle -15.6, 22.061 \rangle$

$|\vec{P_1} + \vec{P_2} + \vec{P_3}| = \sqrt{(-15.6)^2 + 22.061^2} = 27.019\text{N}$

$\theta = \cos^{-1}\left(\frac{-15.6}{27.019}\right) \approx 125.3°$

Vector Word Problems Practice Worksheet 2 (Bearings)

1. An airplane has an airspeed of 500 kph bearing 50°. The wind velocity is 60 kph in the direction of 330°. Find the resultant vector representing the path of the plane relative to the ground. What is the ground speed of the plane? What is the direction?



$\vec{P} = \langle 500\cos 40, 500\sin 40 \rangle = \langle 383.022, 321.394 \rangle$

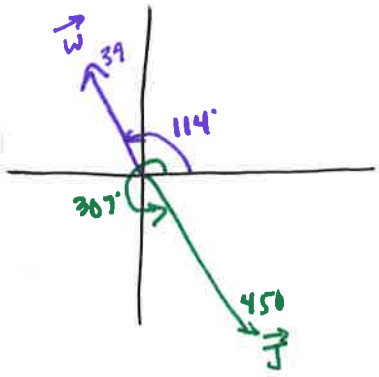
$\vec{w} = \langle 60\cos 330, 60\sin 330 \rangle = \langle 51.962, -30 \rangle$

$\vec{P}+\vec{w} = \langle 434.984, 291.394 \rangle$

$|\vec{P}+\vec{w}| = \sqrt{434.984^2 + 291.394^2} = 523.57 \text{ kph}$

$\theta = \cos^{-1}\left(\frac{434.984}{523.57}\right) = 33.82°$

2. An airline route from San Francisco to Honolulu is S 37° E. A jet flying at 450 mph on that bearing runs into a wind blowing at 39 mph from a direction of 114 degrees. Find the resulting groundspeed and direction of the jet.



$\vec{J} = \langle 450\cos 307°, 450\sin 307° \rangle = \langle 270.817, -359.386 \rangle$

$\vec{w} = \langle 39\cos 114°, 39\sin 114° \rangle = \langle -15.863, 35.628 \rangle$

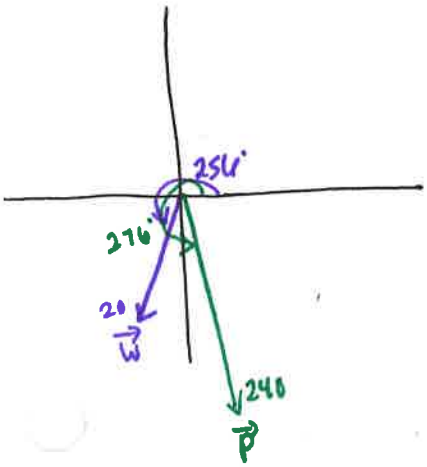
$\vec{J}+\vec{w} = \langle 254.954, -323.758 \rangle$

$|\vec{J}+\vec{w}| = \sqrt{254.954^2 + (-323.758)^2} = 412.093 \text{ mph}$

$\theta = \cos^{-1}\left(\frac{254.954}{412.093}\right) = 51.78 \Rightarrow 360 - 51.78$

$\theta = 308.22°$

3. An airplane is heading S 6° E at an airspeed of 240 kph. A 20 kph wind is blowing from S 14° W. Find the groundspeed and resultant direction of the plane.



$\vec{P} = \langle 240\cos 276, 240\sin 276° \rangle = \langle 25.087, -238.685 \rangle$

$\vec{w} = \langle 20\cos 256°, 20\sin 256° \rangle = \langle -4.838, -19.406 \rangle$

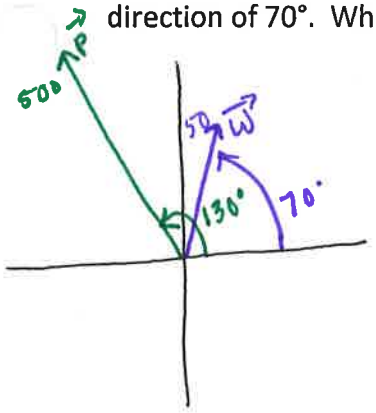
$\vec{P}+\vec{w} = \langle 20.285, -258.091 \rangle$

$|\vec{P}+\vec{w}| = \sqrt{20.285^2 + (-258.091)^2} = 258.884 \text{ kph}$

$\theta = \cos^{-1}\left(\frac{20.285}{258.884}\right) = 85.506°$

$\theta = 360 - 85.506 = 274.494°$

4. An airplane is traveling at a speed of 500 mph with a bearing of 320° at a fixed altitude and no wind. As the plane crosses the Mississippi river, it encounters a wind blowing with a velocity of 50 mph in the direction of 70°. What is the resultant speed and direction of the plane?



$\vec{P} = \langle 500\cos 130, 500\sin 130\rangle = \langle -321.394, 383.022\rangle$

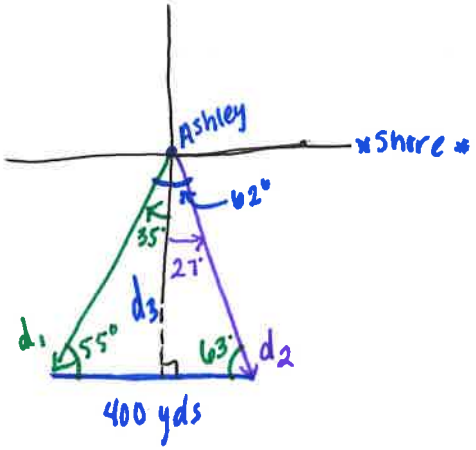
$\vec{W} = \langle 50\cos 70, 50\sin 70\rangle = \langle 17.101, 46.985\rangle$

$\vec{P+W} = \langle -304.293, 430.007\rangle$

$|\vec{P+W}| = \sqrt{(-304.293)^2 + 430.007^2} = 526.783 \text{ mph}$

$\theta = \cos^{-1}\left(\frac{-304.293}{526.783}\right) = 125.29°$

5. Sara is in a boat traveling due west parallel to the shore. At one point Sara sees her friend Ashley on the shore at a bearing of S 35° W. Sara continues west for 400 more yards, where now she sees her friend at a bearing of S 27° E. How far is Sara from Ashley at both points? How far is Sara from the shore?



Law of Sines

$\frac{d_1}{\sin 63°} = \frac{400}{\sin 62°}$

$d_1 = \frac{400\cdot\sin 63°}{\sin 62°}$

$d_1 = 403.65 \text{ yd.}$

$\frac{d_2}{\sin 55°} = \frac{400}{\sin 62°}$

$d_2 = \frac{400\cdot\sin 55°}{\sin 62°}$

$d_2 \approx 371.1 \text{ yd.}$

⊥ distance (d_3)

$\sin 63° = \frac{d_3}{371.1}$

$d_3 = 371.1\cdot\sin 63°$

$d_3 \approx 330.65 \text{ yd}$