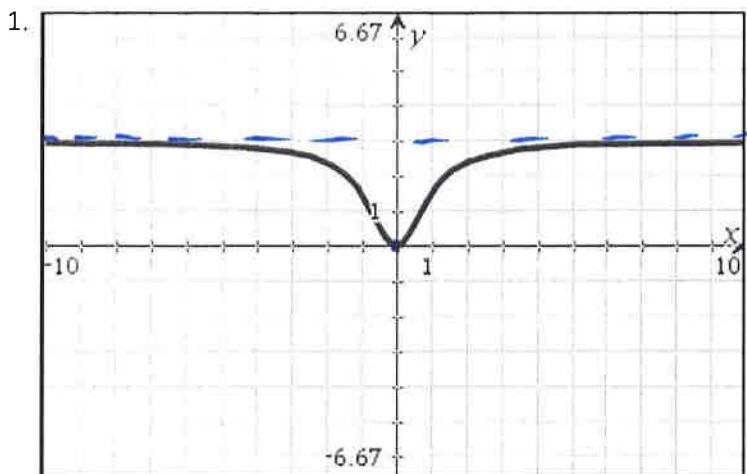


Pre-Calculus: Key Features of Rational Functions

Name KEY

Day 1

Find all key features of the following functions:

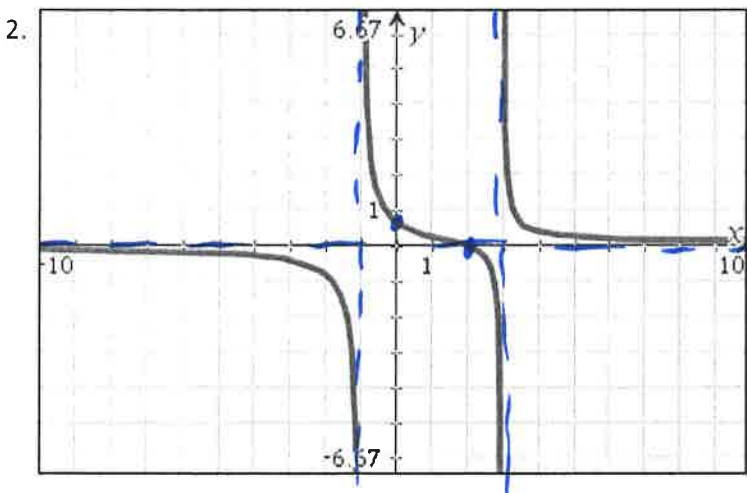


x-intercept(s) (0,0) y-intercept (0,0)

Vertical Asymptote(s) None

End Behavior y = 3

Domain $(-\infty, \infty)$ Range $[0, 3)$

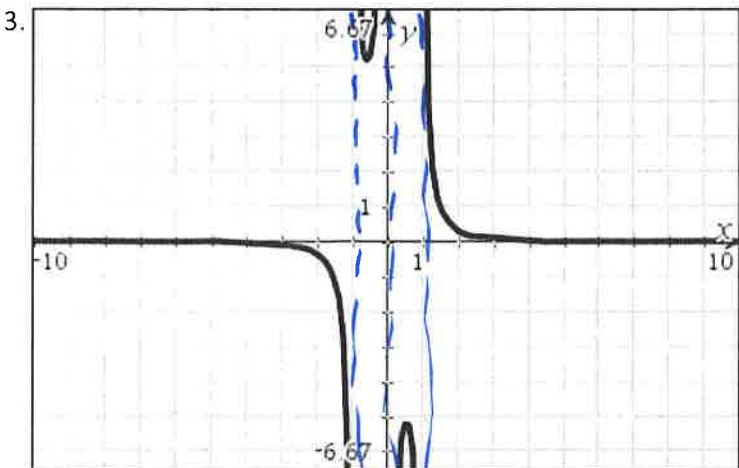


x-intercept(s) (2,0) y-intercept (0, -1.5) ^{guess}

Vertical Asymptote(s) x = -1, x = 3

End Behavior y = 0

Domain $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$ Range $(-\infty, 0) \cup (0, \infty)$

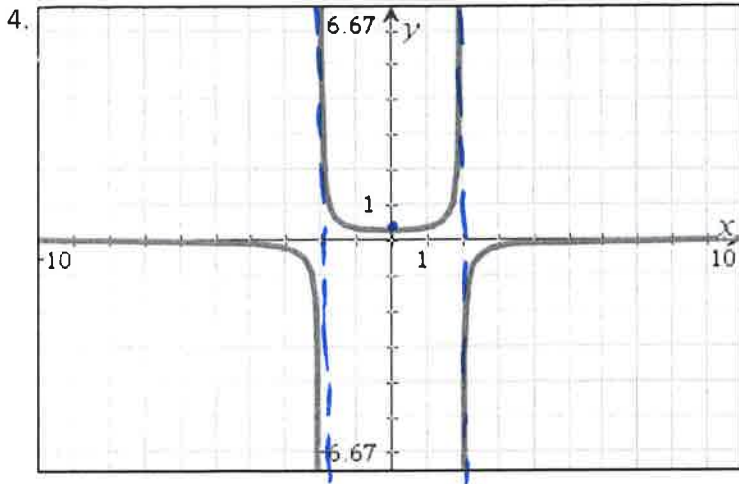


x-intercept(s) None y-intercept None

Vertical Asymptote(s) x = -1, x = 0, x = 1

End Behavior y = 0

Domain $(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$ Range $(-\infty, 0) \cup (0, \infty)$

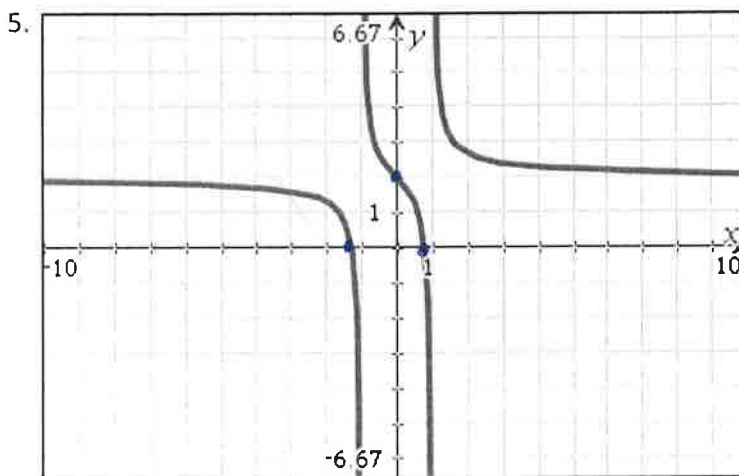


x-intercept(s) None y-intercept (0, 0.25)

Vertical Asymptote(s) $x = -2, x = 2$

End Behavior $y = 0$

Domain $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$ Range $(-\infty, 0) \cup (0, \infty)$

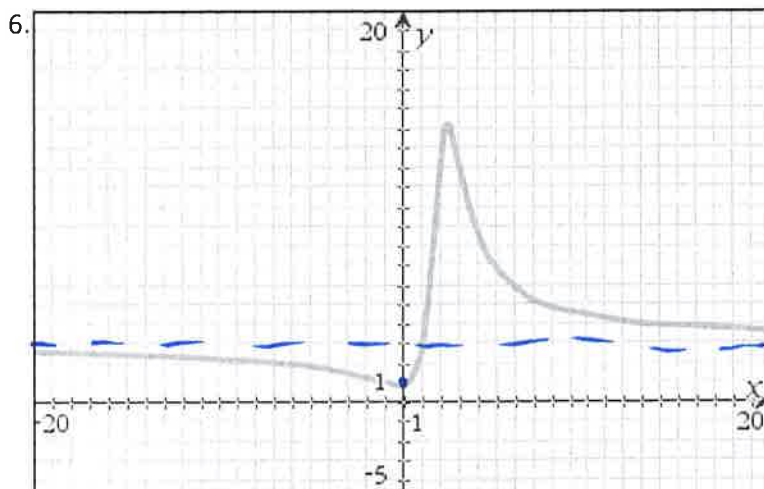


x-intercept(s) $(-1.25, 0)$
 $(0.75, 0)$ y-intercept $(0, 2)$

Vertical Asymptote(s) $x = -1, x = 1$

End Behavior $y = 2$

Domain $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ Range $(-\infty, \infty)$



x-intercept(s) None y-intercept $(0, 1)$

Vertical Asymptote(s) None

End Behavior ~~None~~ $y = 3$

Domain $(-\infty, \infty)$ Range $[0.5, 14)$

Pre-Calculus: Key Features of Rational Functions

Name _____

Day 2

Find all key features of the following functions.

Take off
Domain +
Range

1. $f(x) = \frac{1}{x+3}$

x-intercept(s) None y-intercept $(0, \frac{1}{3})$
 $1 \neq 0$ $f(x) = \frac{1}{0+3}$

Vertical Asymptote(s) $X = -3$
 $X+3=0$

End Behavior $y = 0$
 $n=0$ $n < m$
 $m=1$

Domain $(-\infty, -3) \cup (-3, \infty)$ Range _____
 $X+3 \neq 0$
 $X \neq -3$

2. $f(x) = \frac{2x^2-1}{x^2+3}$

x-intercept(s) $(\frac{\sqrt{2}}{2}, 0)$ $(-\frac{\sqrt{2}}{2}, 0)$ y-intercept $(0, -\frac{1}{3})$
 $2x^2-1=0$ $2x^2=1$ $x = \pm \frac{1}{\sqrt{2}}$ $f(x) = \frac{0-1}{0+3}$

Vertical Asymptote(s) None
 $x^2+3 \neq 0$ $\sqrt{x^2+3}$ imaginary

End Behavior $y = 2$
 $n=2$ $n = m$
 $m=2$

Domain _____ Range _____

3. $f(x) = \frac{x-3}{x^2+3x} = \frac{x-3}{x(x+3)}$

x-intercept(s) $(3, 0)$ y-intercept None und.
 $x-3=0$ $x=3$ $f(x) = \frac{0-3}{0+0} = \frac{-3}{0}$

Vertical Asymptote(s) $X=0, X=-3$
 $X(X+3)=0$

End Behavior $y = 0$
 $n=1$ $n < m$
 $m=2$

Domain _____ Range _____

$$4. f(x) = \frac{x-2}{x^2-2x-3} = \frac{x-2}{(x+1)(x-3)}$$

$$\text{x-intercept(s)} \quad \frac{(2,0)}{x=2}$$

$$\text{y-intercept} \quad \left(0, \frac{2}{3}\right)$$

$$f(x) = \frac{0-2}{0-0-3} = \frac{-2}{-3}$$

$$\text{Vertical Asymptote(s)} \quad x=-1, x=3$$

$$(x+1)(x-3)=0$$

$$\text{End Behavior} \quad y=0$$

$$n=1$$

$$m=2 \quad n < m$$

$$\text{Domain} \quad \text{Range}$$

$$5. f(x) = \frac{-3x^2+x+12}{x^2-4} = \frac{-3x^2+x+12}{(x+2)(x-2)}$$

$$-3x^2+x+12=0$$

$$\frac{-1 \pm \sqrt{1^2 - 4(-3)(12)}}{2(-3)}$$

Need quad. formula
or
graphing calculator ::

$$\text{x-intercept(s)} \quad \frac{(-1.84, 0)}{(2.17, 0)}$$

$$\text{y-intercept} \quad (0, -3)$$

$$f(x) = \frac{0+0+12}{0-4} = \frac{12}{-4}$$

$$\text{Vertical Asymptote(s)} \quad x=-2, x=2$$

$$x^2-4=0$$

$$x^2=4 \quad x=\pm 2$$

$$\text{End Behavior} \quad y = \frac{-3}{1} = -3$$

$$n=2$$

$$m=2 \quad n=m$$

$$\text{Domain} \quad \text{Range}$$

$$6. f(x) = \frac{2x+1}{x^2-x} = \frac{2x+1}{x(x-1)}$$

$$\text{x-intercept(s)} \quad \left(\frac{-1}{2}, 0\right)$$

$$2x+1=0$$

$$\text{y-intercept} \quad \text{undef.}$$

$$f(x) = \frac{0+1}{0-0} = \frac{1}{0} \leftarrow \text{undef.}$$

$$\text{Vertical Asymptote(s)} \quad x=0, x=1$$

$$x(x-1)=0$$

$$\text{End Behavior} \quad y=0$$

$$n=1$$

$$m=2 \quad n < m$$

$$\text{Domain} \quad \text{Range}$$

$$7. f(x) = \frac{x^2 - x - 2}{x^2 - 2x - 8} = \frac{(x+1)(x-2)}{(x+2)(x-4)}$$

x-intercept(s) $(-1, 0)$ $(2, 0)$ y-intercept $(0, \frac{1}{4})$
 $x+1=0$ $x-2=0$ $f(x) = \frac{0-0-2}{0-0-8} = \frac{2}{8}$

Vertical Asymptote(s) $x = -2, x = 4$
 $x+2=0$ $x-4=0$

End Behavior $y = 1$
 $n=2$ $m=2$ $n=m$

Domain _____ Range _____

$$8. f(x) = \frac{2x^5 + x^2 - x + 1}{x^2 - 1}$$

Graph!
 x-intercept(s) $(-1, 1, 0)$ y-intercept $(0, -1)$
 $f(x) = \frac{0+0-0+1}{0-1} = \frac{1}{-1}$

Vertical Asymptote(s) $x = -1, x = 1$
 $x^2 - 1 = 0$ $x = \pm 1$

End Behavior $y = 2x^3 + 2x + 1$
 $n=5$ $m=2$ $n > m \dots$ divide

Domain _____ Range _____

$$\begin{array}{r} 2x^3 + 2x + 1 + \frac{2x+2}{x^2-1} \\ x^2-1 \overline{) 2x^5 + 0x^4 + 0x^3 + x^2 - x + 1} \\ \underline{-(2x^5 - 2x^3)} \\ 2x^3 + x^2 - x + 1 \\ \underline{-(2x^3 - 2x)} \\ x^2 + x + 1 \\ \underline{-(x^2 - x - 1)} \\ 2x + 2 \end{array}$$

Day 3

** Add holes... **

Identify all key features of the rational functions below, then graph those functions.

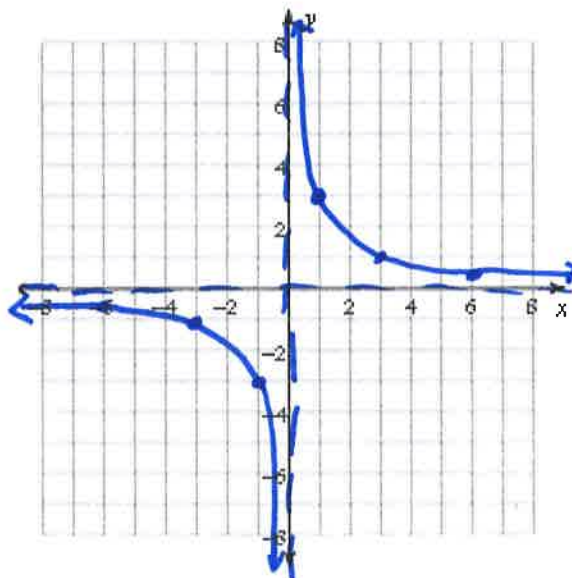
1) $f(x) = \frac{3}{x}$

x-intercept(s) None y-intercept None
 $3 \neq 0$ $\frac{3}{0}$

Vertical Asymptote(s) $x=0$
 $x=0$

End Behavior $y=0$
 $n=0$
 $m=1$ $n < m$

Domain _____ Range _____



Holes? None

x	y
-6	-1/2
-3	-1
-1	-3
1	3

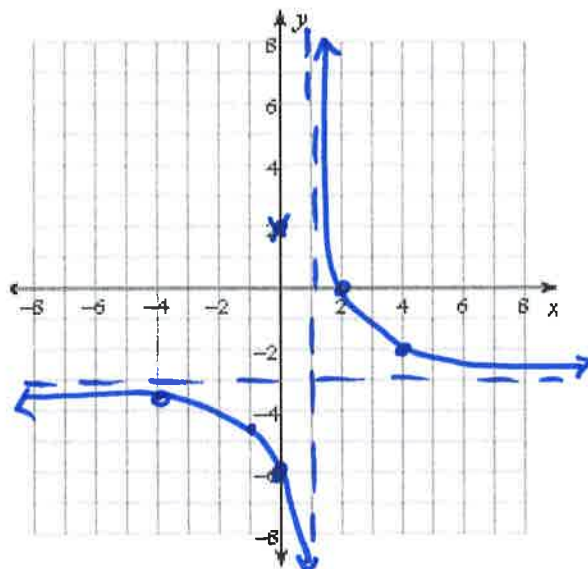
2) $f(x) = \frac{-3x^2 - 6x + 24}{x^2 + 3x - 4} = \frac{-3(x^2 + 2x - 8)}{(x+4)(x-1)} = \frac{-3(x+4)(x-2)}{(x+4)(x-1)}$

x-intercept(s) (2,0) y-intercept (0,-6)
 $x-2=0$ $f(x) = \frac{-3(0-2)}{0-1} = \frac{6}{-1}$
 $x=2$

Vertical Asymptote(s) $x=1$
 $x-1=0$

End Behavior $y=-3$
 $n=2$
 $m=2$ $n=m$

Domain _____ Range _____



Hole!

Holes? $x+4=0$
 $x=-4$

x	y
-1	$\frac{-3(-1-2)}{-1-1} = \frac{9}{-2} = -4.5$
-4	$\frac{-3(-4-2)}{-4-1} = \frac{18}{-5} = -3.6$
4	$\frac{-3(4-2)}{4-1} = \frac{-6}{3} = -2$

$$3) f(x) = \frac{x^3 - 5x^2 + 4x}{3x^2 - 6x - 9} = \frac{x(x^2 - 5x + 4)}{3(x^2 - 2x - 3)} = \frac{x(x+4)(x-1)}{3(x+1)(x-3)}$$

x-intercept(s) $(0,0)$ $(4,0)$ y-intercept $(0,0)$
 $X(x-4)(x-1) = 0$ $(1,0)$ $f(x) = \frac{0-0+0}{0-0-9} = \frac{0}{-9}$

Vertical Asymptote(s) $X = -1$ $X = 3$
 $3(x+1)(x-3) = 0$

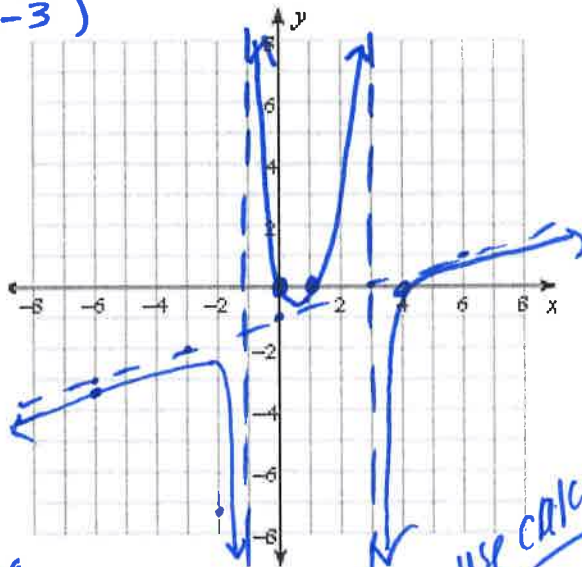
End Behavior $y = \frac{1}{3}x - 1$

$n=3$
 $m=2$ $n > m \dots$ divide

Domain _____ Range _____
 $\frac{1}{3}x - 1$ $r \frac{x-9}{3x^2-6x-9}$

Holes? None

$$\begin{array}{r} 3x^2 - 6x - 9 \overline{) X^3 - 5X^2 + 4X} \\ - X^3 - 2X^2 - 3X \\ \hline -3X^2 + 7X \\ - (-3X^2 + 6X + 9) \\ \hline X - 9 \end{array}$$



use calculator

x	y
-6	$\frac{-6(-6-4)(-6-1)}{3(-6+1)(-6-3)} = \frac{-420}{135} \approx -3.1$
-2	$\frac{-6(-2-4)(-2-1)}{3(-2+1)(-2-3)} = \frac{-108}{15} = -7.2$

4) $f(x) = -\frac{2x}{x^2 - 2x} = -\frac{2x}{x(x-2)}$

x-intercept(s) None y-intercept $(0,1)$
 $\frac{-2}{0-2} = \frac{-2}{-2}$

Vertical Asymptote(s) $X = 2$
 $X - 2 = 0$

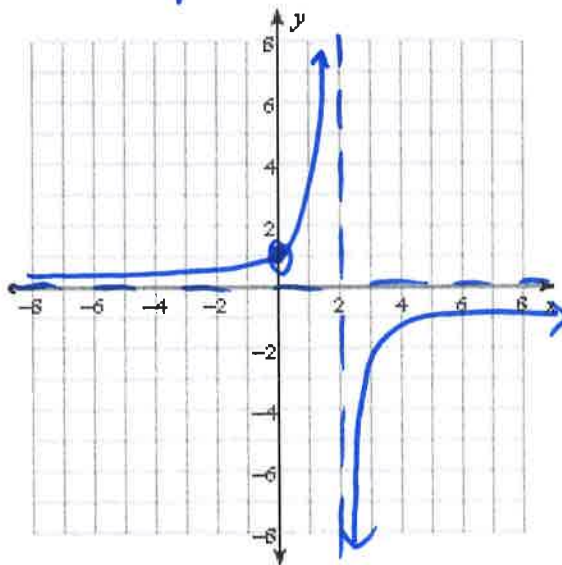
End Behavior $y = 0$

$n=1$
 $m=2$ $n < m$

Domain _____ Range _____

Holes? $X = 0$

x	y
1	$-\frac{2}{1-2} = \frac{-2}{-3} = \frac{2}{3}$



$$5) f(x) = \frac{-x^2 - x + 6}{x^2 - 2x - 3} = \frac{-1(x+3)(x-2)}{(x+1)(x-3)}$$

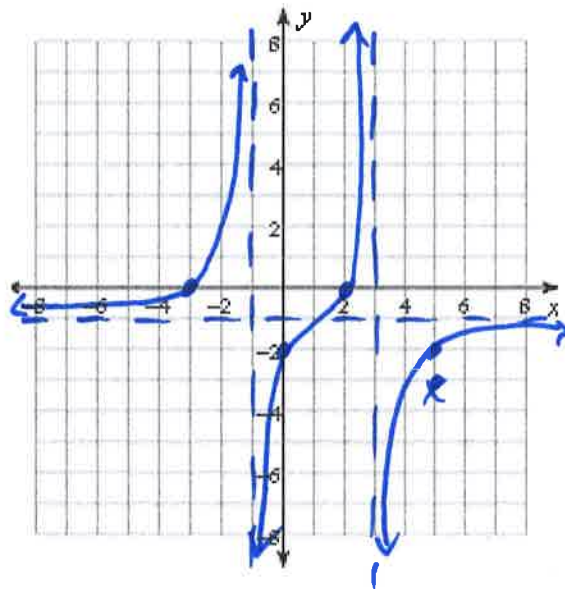
x-intercept(s) $(-3,0)$ $(2,0)$ y-intercept $(0,-2)$
 $x+3=0$ $x-2=0$ $f(x) = \frac{0-0+6}{0-0-3} = \frac{6}{-3}$

Vertical Asymptote(s) $x=-1$ $x=3$
 $x+1=0$ $x-3=0$

End Behavior $y=-1$
 $n=2$
 $m=2$ $n=m$

Domain _____ Range _____

Holes? None



$$6) f(x) = \frac{x^2 - 16}{-3x + 6} = \frac{(x+4)(x-4)}{-3(x-2)}$$

x-intercept(s) $(-4,0)$ $(4,0)$ y-intercept $(0,-2.6)$
 $x+4=0$ $x-4=0$ $f(x) = \frac{0-16}{0+6} = \frac{-16}{6} \approx -2.6$

Vertical Asymptote(s) $x=2$
 $x-2=0$

End Behavior $y = \frac{-1}{3}x - \frac{2}{3}$
 $n=2$ $n > m$
 $m=1$

Domain _____ Range _____

Holes? None

$$\begin{array}{r}
 -\frac{1}{3}x - \frac{2}{3} \\
 \hline
 -3x + 6 \overline{) X^2 + 0X - 16} \\
 \underline{X^2 - 2X} \\
 2X - 16 \\
 \underline{2X - 4} \\
 -12
 \end{array}$$

