Sum and Difference Formulas *Students will use the sum and difference formulas for sines, cosines, and tangents.*	Precalculus

Sum and Difference Identities

$$\sin(u+v)=\sin u\cos v+\cos u\sin v$$

$$\sin(u-v)=\sin u\cos v-\cos u\sin v$$

$$\cos(u+v)=\cos u\cos v-\sin u\sin v$$

$$\cos(u-v)=\cos u\cos v+\sin u\sin v$$

$$\tan(u+v)=\frac{\tan u+\tan v}{1-\tan u\tan v}$$

$$\tan(u-v)=\frac{\tan u-\tan v}{1+\tan u\tan v}$$

When using the sum and difference formulas replace the variables **u** and **v** with angle measures found on the unit circle that will allow you to find the value of trig function of an angle not found on the unit circle.

Example 1: ***Using the Sine/Cosine of a sum or difference to find the exact value.***

a. $\cos 15^\circ = \cos(45-30)$

$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

b. $\sin 75^\circ = \sin(45+30)$

$= \sin 45 \cos 30 + \cos 45 \sin 30^\circ$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

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$\frac{\pi}{4} + \frac{\pi}{3}$

$\frac{3\pi}{12} + \frac{4\pi}{12}$

Example 2: ***Using the Sine/Cosine of a sum or difference to find the exact value.***

a. $\sin\frac{7\pi}{12}$ $= \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

b. $\tan\frac{\pi}{12}$ $= \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 - (\tan\frac{\pi}{3})(\tan\frac{\pi}{4})} = \frac{\sqrt{3} - 1}{1 - \sqrt{3}}$

Example 3: ***Write each expression as the sine, cosine or tangent of an angle. Then find the exact value of the expression.***

a. $\cos 80° \cos 20° + \sin 80° \sin 20°$ $= \cos(80-20)$

$= \cos 60$

$= 1/2$

b. $\cos 25° \cos 20° - \sin 25 \sin 20$ $= \cos(25+20)$

$= \cos 45$

$= \sqrt{2}/2$

8. tan 195°

$$\tan(240-45) = \frac{\tan 240 - \tan 45}{1+\tan 240 \tan 45} = \frac{\sqrt{3}-1}{1+\sqrt{3}}$$

9. Tan 165°

$$\tan(120+45) = \frac{\tan 120 + \tan 45}{1-\tan 120 \tan 45} = \frac{-\sqrt{3}+1}{1-(-\sqrt{3}\cdot 1)} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Write each expression as the sine, cosine or tangent of an angle. Then find the exact value of the expression.

10. sin 25° cos 5° + cos 25° sin 5° $= \sin(25+5) = \sin 30° = \frac{1}{2}$

11. cos 105°cos 45° + sin 105°sin 45° $= \cos(105-45) = \cos 60° = \frac{1}{2}$

12. $\frac{\tan 10°+\tan 35°}{1-\tan 10° \tan 35°} = \tan(10+35) = \tan 45° = 1$

13. $\sin\frac{5\pi}{12}\cos\frac{\pi}{4} - \cos\frac{5\pi}{12}\sin\frac{\pi}{4} = \sin\left(\frac{5\pi}{12}-\frac{\pi}{4}\right) = \sin\left(\frac{5\pi}{12}-\frac{3\pi}{12}\right)$

$= \sin\frac{2\pi}{12} = \sin\frac{\pi}{6} = \frac{1}{2}$

14. $\frac{\tan\frac{\pi}{5} - \tan\frac{\pi}{30}}{1+\tan\frac{\pi}{5}\tan\frac{\pi}{30}} = \tan\left(\frac{\pi}{5}-\frac{\pi}{30}\right) = \tan\left(\frac{6\pi}{30}-\frac{\pi}{30}\right) =$

$\tan\frac{5\pi}{30} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

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Find the exact value of each expression.

1. cos 105°

$$\cos(60+45) = \cos 60 \cos 45 - \sin 60 \sin 45$$
$$= \left(\tfrac{1}{2} \cdot \tfrac{\sqrt{2}}{2}\right) - \left(\tfrac{\sqrt{3}}{2} \cdot \tfrac{\sqrt{2}}{2}\right)$$
$$= \tfrac{\sqrt{2}}{4} - \tfrac{\sqrt{6}}{4} = \boxed{\tfrac{\sqrt{2}-\sqrt{6}}{4}}$$

2. sin 15°

$$\sin(45-30) = \sin 45 \cos 30 - \cos 45 \sin 30$$
$$= \left(\tfrac{\sqrt{2}}{2} \cdot \tfrac{\sqrt{3}}{2}\right) - \left(\tfrac{\sqrt{2}}{2} \cdot \tfrac{1}{2}\right) = \tfrac{\sqrt{6}}{4} - \tfrac{\sqrt{2}}{4} = \boxed{\tfrac{\sqrt{6}-\sqrt{2}}{4}}$$

3. cos 75°

$$\cos(45+30) = \cos 45 \cos 30 - \sin 45 \sin 30$$
$$= \left(\tfrac{\sqrt{2}}{2} \cdot \tfrac{\sqrt{3}}{2}\right) - \left(\tfrac{\sqrt{2}}{2} \cdot \tfrac{1}{2}\right) = \tfrac{\sqrt{6}}{4} - \tfrac{\sqrt{2}}{4} = \boxed{\tfrac{\sqrt{6}-\sqrt{2}}{4}}$$

4. sin 255°

$$\sin(315-60) = \sin 315 \cos 60 - \cos 315 \sin 60$$
$$\left(-\tfrac{\sqrt{2}}{2} \cdot \tfrac{1}{2}\right) - \left(\tfrac{\sqrt{2}}{2} \cdot \tfrac{\sqrt{3}}{2}\right) = -\tfrac{\sqrt{2}}{4} - \tfrac{\sqrt{6}}{4}$$

5. Cos 285°

$$\cos(240+45) = \cos 240^\circ \cos 45^\circ - \sin 240 \sin 45$$
$$\left(-\tfrac{1}{2} \cdot \tfrac{\sqrt{2}}{2}\right) - \left(-\tfrac{\sqrt{3}}{2} \cdot \tfrac{\sqrt{2}}{2}\right)$$
$$-\tfrac{\sqrt{2}}{4} + \tfrac{\sqrt{6}}{4} = \boxed{\tfrac{\sqrt{6}-\sqrt{2}}{4}}$$

6. Cos 195°

$$\cos(240-45)$$
$$= \left(-\tfrac{1}{2} \cdot \tfrac{\sqrt{2}}{2}\right) + \left(-\tfrac{\sqrt{3}}{2} \cdot \tfrac{\sqrt{2}}{2}\right) = \tfrac{-\sqrt{2}-\sqrt{6}}{4}$$

7. Tan 105

$$\tan(60+45) = \frac{\tan 60 + \tan 45}{1 - (\tan 60)(\tan 45)}$$
$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} = \boxed{\frac{\sqrt{3}+1}{1-\sqrt{3}}}$$