

Solving Rational Equations

- Students will be able to find solutions to rational equations.

KEY

RPC/HPC

What is a rational equation and how can it be solved?

$$r(x) = \frac{f(x)}{g(x)} \text{ is a Rational Function}$$

Solving a rational equation

- Guidelines
 - Find the Least Common Denominator for all terms *Goal: Get rid of fractions*
 - Multiply EVERY TERM by the LCD
 - Use "traditional" Algebra solving techniques to solve for the variable
 - CHECK for extraneous solutions

Example 1: Solve $x + \frac{3}{x} = 4$ for x

LCD is x

$$x \left(\frac{x}{1} + \frac{3}{x} = 4 \right)$$

$$x^2 + 3 = 4x$$

$$x^2 - 4x + 3 = 0$$

$$(x - 3)(x - 1) = 0$$

$$x = 3 \quad x = 1$$

$$x = 3 \rightarrow \frac{3}{1} + \frac{3}{3} = 4$$

$$3 + 1 = 4 \checkmark$$

$$x = 1 \rightarrow \frac{1}{1} + \frac{3}{1} = 4$$

$$1 + 3 = 4 \checkmark$$

Example 2: Solve $x + \frac{1}{x-4} = 0$ for x

$$\frac{x-4}{1} \left(\frac{x}{1} + \frac{1}{x-4} = 0 \right)$$

$$x^2 - 4x + 1 = 0$$

a=1 b=-4 c=1

LCD is x-4

$$x = \frac{4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{12}}{2}$$

$$x = 3.73 \quad x = 0.268$$

Check for ex. Sol.

$$3.73 + \frac{1}{3.73-4} = 0$$

$$3.73 + \frac{1}{-0.27} = 0 \checkmark$$

$$0.268 + \frac{1}{0.268-4} = 0$$

$$0.268 + \frac{1}{(-0.268)} = 0 \checkmark$$

* Not factorable, use quad formula

Example 3: Solve $\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{x^2-4x+3}$ for x

LCD is (x-3)(x-1)

Check for ex. Sol.

$$\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{x^2-4x+3}$$

$$(x-1)(x-3) \left(\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{(x-3)(x-1)} \right)$$

$$2x(x-3) + 1(x-1) = 2$$

$$2x^2 - 6x + x - 1 = 2$$

$$2x^2 - 5x - 3 = 0$$

$$-2x^2 - 6x + 1x - 3 = 0$$

$$-2x(x+3) - 1(x+3) = 0$$

$$(-2x-1)(x+3) = 0$$

$$-2x-1=0 \quad -x=-3$$

$$x = -\frac{1}{2} \quad x = 3$$

$$\frac{2(3)}{3-1} + \frac{1}{3-3} = \frac{2}{3^2-4(3)+3}$$

x=3 is extraneous

$$\frac{2(-\frac{1}{2})}{-\frac{1}{2}-1} + \frac{1}{-\frac{1}{2}-3} = \frac{2}{(-\frac{1}{2})^2-4(-\frac{1}{2})+3} \checkmark$$

~~WAAA~~

Example 4: Solve $\frac{x-3}{x} + \frac{3}{x+2} + \frac{6}{x^2+2x} = 0$ for x

LCD is x(x+2)

$$x(x+2) \left(\frac{x-3}{x} + \frac{3}{x+2} + \frac{6}{x(x+2)} = 0 \right)$$

$$(x+2)(x-3) + 3x + 6 = 0$$

$$x^2 - 3x + 2x - 6 + 3x + 6 = 0$$

$$x^2 + 2x = 0$$

$$x(x+2) = 0$$

$$x=0 \quad x=-2$$

No Solutions because f(x) will be undefined

HPC/RPC: Solving Rational Functions

Name Key

Solve the following equations. Identify any extraneous solutions.

$$1. \left(x+5 = \frac{14}{x} \right)$$

$$x^2 + 5x = 14$$

$$x^2 + 5x - 14 = 0$$

$$(x+7)(x-2) = 0$$

$x = -7 \quad x = 2$

$$-7+5 = \frac{14}{-7}$$

$$-2 = -2 \checkmark$$

$$2+5 = \frac{14}{2}$$

$$7 = 7 \checkmark$$

$$2. \left(\frac{3}{x-1} + \frac{2}{x} = 8 \right)$$

$$3x + 2x - 2 = 8x^2 - 8x$$

$$0 = 8x^2 - 13x + 2$$

$$a=8 \quad b=-13 \quad c=2$$

$$x = \frac{13 \pm \sqrt{13^2 - 4(8)(2)}}{2(8)}$$

$x = \frac{13 \pm \sqrt{105}}{16}$

 $x = 1.45$
 $x = 0.172$

$$3. \left(\frac{x}{1} + \frac{12}{x} = \frac{7}{1} \right)$$

$$x^2 + 12 = 7x$$

$$x^2 - 7x + 12 = 0$$

$$(x-4)(x-3) = 0$$

$x = 4 \quad x = 3$

$$4 + \frac{12}{4} = 7$$

$$4 + 3 = 7 \checkmark$$

$$3 + \frac{12}{3} = 7$$

$$3 + 4 = 7 \checkmark$$

$$4. 2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$$

$$x(x+4) \left(2 - \frac{3}{x+4} = \frac{12}{x(x+4)} \right)$$

$$2x^2 + 8x - 3x = 12$$

$$2x^2 + 5x - 12 = 0$$

$$2x^2 + 8x - 3x - 12 = 0$$

$$2x(x+4) - 3(x+4) = 0$$

$$2x-3=0 \quad x+4=0$$

$x = 3/2$

 ~~$x = -4$~~

$$x = \frac{-5 + \sqrt{121}}{4} \quad x = \frac{-5 - \sqrt{121}}{4}$$
 ~~$x = 3/2$~~
 ~~$x = -4$~~

~~$\frac{-24}{8} = -3$~~
 $\frac{5}{5}$

$2 - \frac{3}{-4+4} = \frac{12}{-4^2+4(-4)}$
 $x = -4$ is extraneous

→ Quad Form.
 $x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-12)}}{2(2)}$

$$5. \left(\frac{x-3}{x} - \frac{3}{x+1} + \frac{3}{x^2+x} = 0 \right) x(x+1)$$

$$(x+1)(x-3) - 3x + 3 = 0$$

$$x^2 - 3x + x - 3 - 3x + 3 = 0$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

~~$x = 0$~~

$x = 5$

↑
Extraneous

$$6. \left(\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x^2+3x} \right) x(x+3)$$

$$(x+3)(x+3) - 2x = 6$$

$$x^2 + 6x + 9 - 2x - 6 = 0$$

$$x^2 + 4x + 3 = 0$$

$$(x+3)(x+1) = 0$$

~~$x = 3$~~

$x = -1$

↑
Extraneous