

## Solving "Basic" Trigonometric Equations

Honors Precalculus

Students will be able to solve trigonometric equations.

What steps are needed to solve Trig equations?

Use Algebra techniques to isolate the Trig function:

- Add/Subtract/Multiply/Divide both sides
  - By NUMBERS; NOT trig functions
- Factor
- Use the quadratic formula
  - $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Square root both sides
- ETC.

Then: Use an inverse trig function ( $\sin^{-1}$ ;  $\cos^{-1}$ ;  $\tan^{-1}$ ) to solve for the angle(s)

(Or recognize values of trig functions from the Unit Circle)

- Often times there are many angles that are solutions 😊

Example 1:

Solve  $2\sin x = 1$  for all angle values of  $x$  on the interval  $[0, 2\pi)$

$$\begin{aligned} 2\sin x &= 1 \\ \sin x &= \frac{1}{2} \end{aligned} \quad \begin{aligned} x &= \frac{\pi}{6}, \frac{5\pi}{6} \\ &30^\circ, 150^\circ \end{aligned}$$

Example 2:

Solve  $\cos 2x + \sqrt{3} = -\cos 2x$  for all angle values of  $x$  on the interval  $[0, 2\pi)$

$$\begin{aligned} \cos 2x + \sqrt{3} &= -\cos 2x \\ \hline 2\cos 2x + \sqrt{3} &= 0 \\ 2\cos 2x &= -\sqrt{3} \\ \cos 2x &= \frac{-\sqrt{3}}{2} \end{aligned} \quad \begin{aligned} 2x &= \frac{5\pi}{6} & x &= \frac{5\pi}{12} \\ &(150^\circ) & &(75^\circ) \\ 2x &= \frac{7\pi}{6} & x &= \frac{7\pi}{12} \\ &(210^\circ) & &(105^\circ) \end{aligned}$$

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**Example 3:** Solve  $3\cot^2 x - 1 = 0$  for all values of  $x$

$$3\cot^2 x - 1 = 0$$

$$3\cot^2 x = 1$$

$$\cot^2 x = \frac{1}{3}$$

$$\text{so } \left\{ \begin{array}{l} \cot x = \pm \frac{1}{\sqrt{3}} \\ \tan x = \pm \sqrt{3} \end{array} \right.$$

$$x = \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\} + 2n\pi$$

**Example 4:**

Solve  $2\sin^2 x - \sin x - 1 = 0$  for all values of  $x$

$$2a^2 - a - 1$$

if  $a = \sin x$

$$(2\sin x + 1)(\sin x - 1) = 0 \quad \leftarrow (2a + 1)(a - 1) = 0$$

$$2\sin x + 1 = 0$$

$$\sin x - 1 = 0$$

$$2\sin x = -1$$

$$\sin x = 1$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{\pi}{2} + 2n\pi$$

$$x = \left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\} + 2n\pi$$

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### Example 5:

Solve  $\sqrt{2}\csc x + 2 = 4$  for all values of  $x$

$$\sqrt{2}\csc x = 2$$

so...

$$\csc x = \frac{2}{\sqrt{2}}$$

$$\sin x = \frac{\sqrt{2}}{2}$$

$$x = \left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\} + 2n\pi$$

### Example 6:

Solve  $\sec^2 x = \frac{4}{3}$  for all values of  $x$

so

$$\sec x = \frac{2}{\sqrt{3}}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \left\{ \frac{\pi}{6}, \frac{7\pi}{6} \right\} + 2n\pi$$

Solving "simple" trigonometric equations: Practice

Name Key

Solve the following equations for  $x$  over the interval  $[0, 2\pi)$ .

1.  $2 \cos x + 4 = 5$

$$\begin{aligned} 2 \cos x &= 1 \\ \cos x &= \frac{1}{2} \\ x &= \frac{\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

2.  $2 \sin x - 1 = 0$

$$\begin{aligned} 2 \sin x &= 1 \\ \sin x &= \frac{1}{2} \\ x &= \frac{\pi}{6}, \frac{5\pi}{6} \end{aligned}$$

3.  $\tan^2 x - 3 = 0$

$$\begin{aligned} \tan^2 x &= 3 \\ \tan x &= \pm \sqrt{3} \quad \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \\ x &= \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

4.  $5 \cos x - \sqrt{3} = 3 \cos x$

$$\begin{aligned} 2 \cos x - \sqrt{3} &= 0 \\ 2 \cos x &= \sqrt{3} \\ \cos x &= \frac{\sqrt{3}}{2} \\ x &= \frac{\pi}{6}, \frac{11\pi}{6} \end{aligned}$$

5.  $4 \csc^2 x - 2 = 0$

$$\begin{aligned} 4 \csc^2 x &= 2 \\ \csc^2 x &= \frac{1}{2} \\ \csc x &= \pm \frac{1}{\sqrt{2}} \\ \sin x &= \pm \sqrt{2} \quad \text{NO soln} \end{aligned}$$

6.  $4 \sin^2 x - 2 = 0$

$$\begin{aligned} 4 \sin^2 x &= 2 \\ \sin^2 x &= \frac{1}{2} \\ \sin x &= \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2} \\ x &= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \end{aligned}$$

Solve the following equations for all angle values of  $x$ .

7.  $3 \tan x - 3 = 0$

$$\begin{aligned} 3 \tan x &= 3 \\ \tan x &= 1 \\ x &= \frac{\pi}{4}, \frac{5\pi}{4} \} + 2n\pi \end{aligned}$$

8.  $\sec\left(\frac{3x}{2}\right) + 2 = 0$

$$\begin{aligned} \sec \frac{3x}{2} &= -2 \\ \cos \frac{3x}{2} &= -\frac{1}{2} \\ \frac{3x}{2} &= \frac{2\pi}{3}, \frac{4\pi}{3} \\ x &= \frac{4\pi}{9}, \frac{8\pi}{9} \end{aligned}$$

9.  $\sin^2 x - 4 \sin x - 5 = 0$

$$\begin{aligned} (\sin x - 5)(\sin x + 1) &= 0 \\ \sin x = 5 \quad \sin x = -1 \\ x = \text{NO soln} \quad x &= \frac{3\pi}{2} \} + 2n\pi \end{aligned}$$

10.  $5 \cos 2x + 1 = 3 \cos 2x$

$$\begin{aligned} 2 \cos 2x + 1 &= 0 \\ 2 \cos 2x &= -1 \\ \cos 2x &= -\frac{1}{2} \\ 2x &= \frac{2\pi}{3}, \frac{4\pi}{3} \\ x &= \frac{\pi}{3}, \frac{2\pi}{3} \} + 2n\pi \end{aligned}$$

11.  $16 \cos^2 x - 8 = 0$

$$\begin{aligned} 16 \cos^2 x &= 8 \\ \cos^2 x &= \frac{1}{2} \\ \cos x &= \pm \frac{1}{\sqrt{2}} \\ \cos x &= \pm \frac{\sqrt{2}}{2} \\ x &= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \} + 2n\pi \end{aligned}$$

12.  $2 \cos^2 x - 3 \cos x + 1 = 0$

$$\begin{aligned} (2 \cos x - 1)(\cos x - 1) &= 0 \\ \cos x = \frac{1}{2} \quad \cos x = 1 \\ x &= \frac{\pi}{3}, \frac{5\pi}{3}, 0 \} + 2n\pi \end{aligned}$$

# Solving Trigonometric Equations

Using Trig Identities

Name: \_\_\_\_\_ Period: \_\_\_\_\_

Solve for  $x$  over the interval  $[0, 2\pi)$ .

1.  $2\cos^2 x + 3\sin x = 0$

$$2(1 - \sin^2 x) + 3\sin x = 0$$

$$2 - 2\sin^2 x + 3\sin x = 0$$

$$2\sin^2 x - 3\sin x - 2 = 0$$

$$(2\sin x + 1)(\sin x - 2) = 0$$

$$2\sin x + 1 = 0 \quad \sin x - 2 = 0$$

$$\sin x = -\frac{1}{2}$$

$$\sin x = 2 \quad \text{No sol.}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

2.  $2\cos^2 x - \sin x - 1 = 0$

$$2(1 - \sin^2 x) - \sin x - 1 = 0$$

$$2 - 2\sin^2 x - \sin x - 1 = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$2\sin x - 1 = 0 \quad \sin x = -1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

3.  $\sin^2 x - 2\cos x - 2 = 0$

$$1 - \cos^2 x - 2\cos x - 2 = 0$$

$$\cos^2 x + 2\cos x + 1 = 0$$

$$(\cos x + 1)(\cos x + 1) = 0$$

$$\cos x + 1 = 0$$

$$\cos x = -1$$

$$x = \pi$$

4.  $4\sin^2 x + 4\cos x - 5 = 0$

$$4(1 - \cos^2 x) + 4\cos x - 5 = 0$$

$$4 - 4\cos^2 x + 4\cos x - 5 = 0$$

$$4\cos^2 x - 4\cos x + 1 = 0$$

$$(2\cos x - 1)(2\cos x - 1) = 0$$

$$2\cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

5.  $\csc^2 x - 2\cot x = 0$

$$\cot^2 x + 1 - 2\cot x = 0$$

$$\cot^2 x - 2\cot x + 1 = 0$$

$$(\cot x - 1)(\cot x - 1) = 0$$

$$\cot x - 1 = 0$$

$$\cot x = 1$$

$$\rightarrow \tan x = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

6.  $2\tan^2 x - 3\sec x + 3 = 0$

~~Atan x~~

$$2(\sec^2 x - 1) - 3\sec x + 3 = 0$$

$$2\sec^2 x - 2 - 3\sec x + 3 = 0$$

$$2\sec^2 x - 3\sec x + 1 = 0$$

$$(2\sec x - 1)(\sec x - 1) = 0$$

$$2\sec x - 1 = 0$$

$$\sec x = \frac{1}{2}$$

$$\rightarrow \cos x = 2$$

No sol.

$$\sec x - 1 = 0$$

$$\sec x = 1$$

$$\rightarrow \cos x = 1$$

$$x = 0$$

$$7. \sin^2 x - \tan x \cos^2 x = 0$$

$$\sin^2 x - \frac{\sin x}{\cos x} \cdot \frac{\cos^2 x}{1} = 0$$

$$\sin^2 x - \sin x \cos x = 0$$

$$\sin x (\sin x - \cos x) = 0$$

$$\sin x = 0 \quad \sin x - \cos x = 0$$

$$X = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4} \quad \sin x = \cos x$$

$$8. 4\cos^2 x = 5 - 4\sin x$$

$$4(1 - \sin^2 x) + 4\sin x - 5 = 0$$

$$4 - 4\sin^2 x + 4\sin x - 5 = 0$$

$$4\sin^2 x - 4\sin x + 1 = 0$$

$$(2\sin x - 1)(2\sin x - 1) = 0$$

$$2\sin x - 1 = 0$$

$$\sin x = \frac{1}{2}$$

$$X = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$9. \tan^4 x - 2 = \tan^2 x + \sec^2 x$$

$$\cancel{\tan^4 x} - \cancel{\tan^2 x} \tan^4 x - 2 = \tan^2 x + 1 + \tan^2 x$$

$$\tan^4 x - 2 = 2\tan^2 x + 1$$

$$\tan^4 x - 2\tan^2 x - 3 = 0$$

$$(\tan^2 x + 1)(\tan^2 x - 3) = 0$$

$$\tan^2 x = -1$$

$$\tan x = \pm\sqrt{-1}$$

NO SOL.

$$\tan^2 x = 3$$

$$\tan x = \pm\sqrt{3}$$

$$X = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$10. \cos x - \cot x = 0$$

$$\cos x - \frac{\cos x}{\sin x} = 0$$

$$\cos x \left(1 - \frac{1}{\sin x}\right) = 0$$

$$\cos x (1 - \csc x) = 0$$

$$\cos x = 0$$

$$1 - \csc x = 0$$

$$\csc x = 1$$

$$X = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$11. \cos^2 x - \tan x \cos^2 x = 0$$

$$\cos^2 x (1 - \tan x) = 0$$

$$\cos^2 x = 0 \quad 1 - \tan x = 0$$

$$\cos x = 0 \quad \tan x = 1$$

$$X = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}$$

$$12. \sqrt{3}\tan x \sec x + 2\tan x = 0$$

$$\tan x (\sqrt{3}\sec x + 2) = 0$$

$$\tan x = 0 \quad \sqrt{3}\sec x + 2 = 0$$

$$\sec x = -\frac{2}{\sqrt{3}}$$

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$X = 0, \pi, \frac{5\pi}{6}, \frac{7\pi}{6}$$

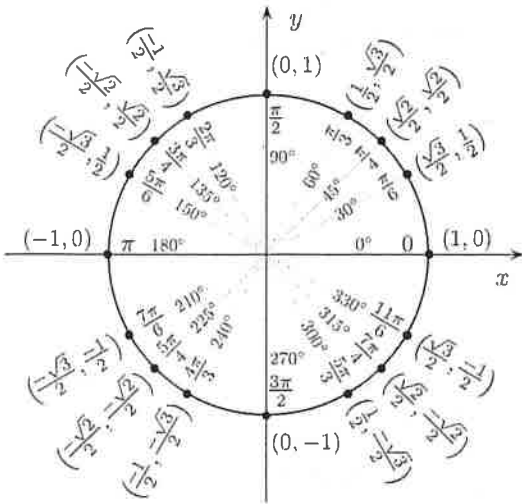
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Practice Worksheet: Trigonometric Equations

Solve each equation over  $[0, 2\pi]$  by combining like terms. All answers must be exact in terms of pi.



<p>1] <math>\sin x + 2 = 3</math>  <math>\sin x = 1</math>  <math>x = \frac{\pi}{2}</math></p>	<p>4] <math>\sqrt{3} \tan x + 1 = 0</math>  <math>\sqrt{3} \tan x = -1</math>  <math>\tan x = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}</math>  <math>x = \frac{5\pi}{6}, \frac{11\pi}{6}</math></p>
<p>2] <math>4 \sec x + 8 = 0</math>  <math>4 \sec x = -8</math>  <math>\sec x = \frac{-2}{1} \rightarrow \cos x = \frac{1}{2}</math>  <math>x = \frac{2\pi}{3}, \frac{4\pi}{3}</math></p>	<p>5] <math>\cot x - \sqrt{3} = 0</math>  <math>\cot x = \sqrt{3}</math>  <math>x = \frac{\pi}{6}, \frac{7\pi}{6}</math></p>
<p>3] <math>18 \cos x - 9\sqrt{3} = 0</math>  <math>18 \cos x = 9\sqrt{3}</math>  <math>\cos x = \frac{\sqrt{3}}{2}</math>  <math>x = \frac{\pi}{6}, \frac{11\pi}{6}</math></p>	<p>6] <math>8 \cos x - 4\sqrt{2} = 0</math>  <math>8 \cos x = 4\sqrt{2}</math>  <math>\cos x = \frac{\sqrt{2}}{2}</math>  <math>x = \frac{\pi}{4}, \frac{7\pi}{4}</math></p>

Solve each equation over  $[0, 2\pi]$  with the square root method. All answers must be exact in terms of pi.

<p>7] <math>\sec^2 x - 1 = 0</math>  <math>\sqrt{\sec^2 x} = \sqrt{1}</math>  <math>\sec x = \pm 1 \rightarrow \cos x = \pm \frac{1}{1}</math>  <math>x = 0, 2\pi, \pi, \pi</math></p>	<p>10] <math>4 \cos^2 x - 1 = 0</math>  <math>4 \cos^2 x = 1</math>  <math>\sqrt{\cos^2 x} = \sqrt{\frac{1}{4}}</math>  <math>\cos x = \pm \frac{1}{2}</math>  <math>x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}</math></p>
<p>8] <math>2 \cos^2 x = 1</math>  <math>\sqrt{\cos^2 x} = \sqrt{\frac{1}{2}}</math>  <math>\cos x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}</math>  <math>x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}</math></p>	<p>11] <math>4 \sin^2 x + 5 = 6</math>  <math>4 \sin^2 x = 1</math>  <math>\sqrt{\sin^2 x} = \sqrt{\frac{1}{4}}</math>  <math>\sin x = \pm \frac{1}{2}</math>  <math>x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}</math></p>
<p>9] <math>3 \tan^2 x - 9 = 0</math>  <math>3 \tan^2 x = 9</math>  <math>\sqrt{\tan^2 x} = \sqrt{3}</math>  <math>\tan x = \pm \sqrt{3}</math>  <math>x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}</math></p>	<p>12] <math>3 \sec^2 x - 4 = 0</math>  <math>3 \sec^2 x = 4</math>  <math>\sqrt{\sec^2 x} = \sqrt{\frac{4}{3}}</math>  <math>\sec x = \pm \frac{2}{\sqrt{3}} \rightarrow \cos x = \pm \frac{\sqrt{3}}{2}</math>  <math>x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}</math></p>

Solve each equation over  $[0, 2\pi]$  by factoring. All answers must be exact in terms of  $\pi$ .

<p>13] <math>\sin^2 x - 3 \sin x + 2 = 0</math>  <math>(\sin x - 2)(\sin x - 1) = 0</math>  <math>\sin x = 2</math>    <math>\sin x = 1</math>  <math>x = \text{undefined}</math>    <math>x = \frac{\pi}{2}</math></p>	<p>17] <math>\cot^2 x = -2 \cot x - 1</math>  <math>\cot^2 x + 2 \cot x + 1 = 0</math>  <math>(\cot x + 1)(\cot x + 1) = 0</math>  <math>\cot x = -1</math>  <math>x = \frac{3\pi}{4}, \frac{7\pi}{4}</math></p>
<p>14] <math>\sin^2 x \cos x = \cos x</math>  <math>\sin^2 x \cos x - \cos x = 0</math>  <math>\cos x (\sin^2 x - 1) = 0</math>  <math>\cos x = 0</math>    <math>\sqrt{\sin^2 x} = \sqrt{1}</math>  <math>\sin x = \pm 1</math>  <math>x = \frac{\pi}{2}, \frac{3\pi}{2}</math>    <math>x = \frac{\pi}{2}, \frac{3\pi}{2}</math></p>	<p>18] <math>\sin x - 2 \sin x \cos x = 0</math>  <math>\sin x (1 - 2 \cos x) = 0</math>  <math>\sin x = 0</math>    <math>1 - 2 \cos x = 0</math>  <math>\phantom{\sin x = 0}</math>    <math>-2 \cos x = -1</math>  <math>\phantom{\sin x = 0}</math>    <math>\cos x = \frac{1}{2}</math>  <math>x = 0, \pi, 2\pi</math>    <math>x = \frac{\pi}{3}, \frac{5\pi}{3}</math></p>
<p>15] <math>2 \cos^2 x - \sqrt{3} \cos x = 0</math>  <math>\cos x (2 \cos x - \sqrt{3}) = 0</math>  <math>\cos x = 0</math>    <math>2 \cos x - \sqrt{3} = 0</math>  <math>\phantom{\cos x = 0}</math>    <math>2 \cos x = \sqrt{3}</math>  <math>\phantom{\cos x = 0}</math>    <math>\cos x = \frac{\sqrt{3}}{2}</math>  <math>x = \frac{\pi}{2}, \frac{3\pi}{2}</math>    <math>x = \frac{\pi}{6}, \frac{11\pi}{6}</math></p>	<p>19] <math>\sec x \csc x = 2 \csc x</math>  <math>\sec x \csc x - 2 \csc x = 0</math>  <math>\csc x (\sec x - 2) = 0</math>  <math>\csc x = 0</math>    <math>\sec x - 2 = 0</math>  <math>\rightarrow \sin x = \frac{1}{0}</math>    <math>\sec x = 2</math>  <math>\phantom{\rightarrow \sin x = \frac{1}{0}}</math>    <math>\rightarrow \cos x = \frac{1}{2}</math>  <math>x = \text{undefined}</math>    <math>x = \frac{\pi}{3}, \frac{5\pi}{3}</math></p>
<p>16] <math>2 \sin^2 x + \sin x = 1</math>  <math>2 \sin^2 x + \sin x - 1 = 0</math>  <math>(2 \sin x - 1)(\sin x + 1) = 0</math>  <math>2 \sin x - 1 = 0</math>    <math>\sin x + 1 = 0</math>  <math>\sin x = \frac{1}{2}</math>    <math>\sin x = -1</math>  <math>x = \frac{\pi}{6}, \frac{5\pi}{6}</math>    <math>x = \frac{3\pi}{2}</math></p>	<p>20] <math>\tan x \csc x - 2 \tan x = 0</math>  <math>\tan x (\csc x - 2) = 0</math>  <math>\tan x = 0</math>    <math>\csc x = 2</math>  <math>x = 0, \pi, 2\pi</math>    <math>\rightarrow \sin x = \frac{1}{2}</math>  <math>\phantom{x = 0, \pi, 2\pi}</math>    <math>x = \frac{\pi}{6}, \frac{5\pi}{6}</math></p>