

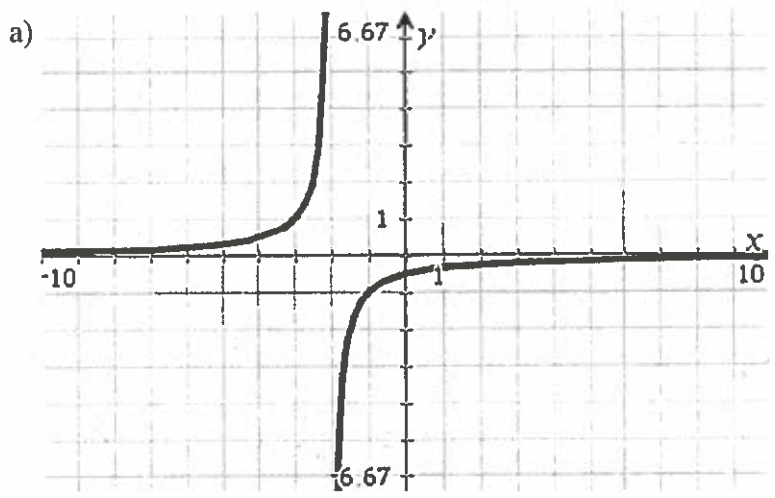
HPC/RPC Review
Rational Functions

Name Key

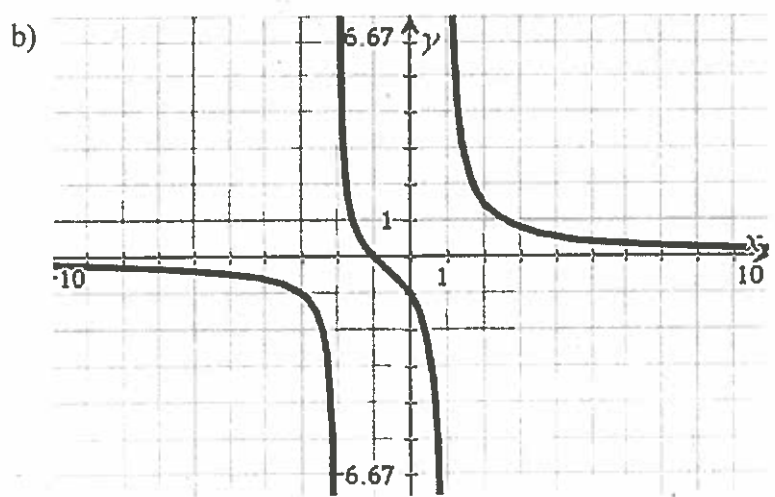
Date _____ Period _____

PC Reporting Strand: Functions (Identify key features of graphs) Score _____

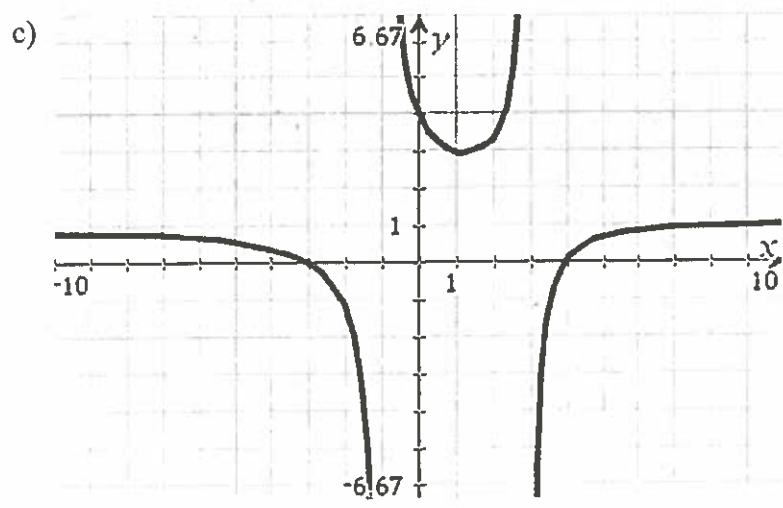
1. Given the graphs of rational functions below, identify all key features of each.



x-intercept(s): None
 y-intercept: $-\frac{1}{2}$ $(0, -\frac{1}{2})$
 Vertical Asymptote(s): $x = -2$
 End Behavior: $y = 0$
 Asympmtote



x-intercept(s): -1 $(-1, 0)$
 y-intercept: -1 $(0, -1)$
 Vertical Asymptote(s): $x = -2, x = 1$
 End Behavior: $y = 0$
 Asympmtote



x-intercept(s): $-3, 4$ $(-3, 0)$ $(4, 0)$
 y-intercept: 4 $(0, 4)$
 Vertical Asymptote(s): $x = -1, x = 3$
 End Behavior: $y = 1$
 Asympmtote

2. Given the equations below, identify all key features of the rational function.

a) $f(x) = \frac{3}{x+4}$

x-intercept(s): None
 $3 \neq 0$

y-intercept: $y = \frac{3}{4}$

$y = \frac{3}{0+4} = \frac{3}{4}$

Vertical Asymptote(s): $X = -4$
 $X+4=0$

Hole(s): None

End Behavior Asymptote: $y=0$

$n=0$ $m=1$ $n < m$

b) $f(x) = \frac{4x-12}{x^3-x^2-6x}$

x-intercept(s): None
 $4 \neq 0$

y-intercept: None

$f(x) = \frac{4(x-3)}{x(x+2)(x-3)}$

$y = \frac{0-12}{0-0-0} = 0$ in denominator

Vertical Asymptote(s): $X=0, X=-2$
 $X=0$ $X+2=0$

Hole(s): $X=3$
 $X-3=0$

End Behavior Asymptote: $y=0$

$n=1$ $m=3$
 $n < m$

c) $f(x) = -\frac{1}{x-4} + 1$

x-intercept(s): 5 (5,0)
 $X-5=0$
 $X=5$

y-intercept: $\frac{5}{4}$ (0, $\frac{5}{4}$)

$\frac{-1}{x-4} + \frac{x-4}{x-4} = \frac{x-5}{x-4}$

$y = \frac{0-5}{0-4} = \frac{-5}{-4}$

Vertical Asymptote(s): $X=4$
 $X-4=0$

Hole(s): None

End Behavior Asymptote: $y=1$

$n=1$ $m=1$ $n=m$

Tough Factor

$$d) f(x) = \frac{x^3 + 4x^2 - x - 4}{x+1} = \frac{x^2(x+4) - 1(x+4)}{x+1} = \frac{(x^2-1)(x+4)}{x+1} = \frac{(x-1)(x+4)}{x+1}$$

x-intercept(s): 1, -4 (1,0) (-4,0)
 $x-1=0$ $x+4=0$

y-intercept: $y = -4$ (0, -4)
 $y = \frac{0+0-0-4}{0+1} = -4$

Vertical Asymptote(s): None

Hole(s): $x = -1$
 $x+1=0$

End Behavior Asymptote: $y = x^2 + 3x - 4$
 $n=3$ $m=1$ ($n > m$)

Synthetic Division

$$\begin{array}{r|rrrr} -1 & 1 & 4 & -1 & -4 \\ & & -1 & -3 & 4 \\ \hline & 1 & 3 & -4 & 0 \end{array}$$

3. Given the equations below, identify all key features of the rational function, then graph it.

$$a) f(x) = \frac{-4x-4}{x^2+x-2} = \frac{-4(x+1)}{(x+2)(x-1)}$$

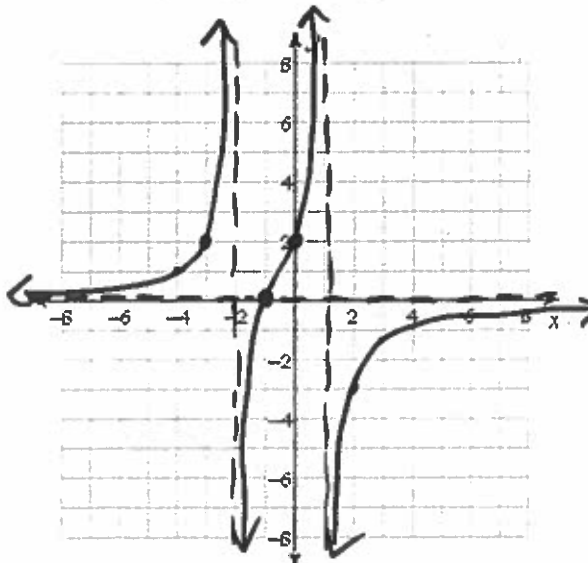
x-intercept(s): -1 (-1,0)
 $-4x-4=0$
 $x=-1$

y-intercept: $y = 2$
 $y = \frac{0-4}{0+0-2} = \frac{-4}{-2}$

Vertical Asymptote(s): $x = -2, x = 1$
 $x+2=0$ $x-1=0$

Hole(s): None

End Behavior Asymptote: $y = 0$
 $n=1$ $m=2$ $n < m$



$$2(x^2 - x - 6) \quad b) f(x) = \frac{x^2 - 9}{2x^2 - 2x - 12} = \frac{(x+3)(x-3)}{2(x+2)(x+3)}$$

x-intercept(s): $x = -3$
 $x + 3 = 0$

y-intercept: $\frac{3}{4}$

$$y = \frac{0 - 9}{0 - 0 - 12} = \frac{-9}{-12}$$

Vertical Asymptote(s): $x = -2$

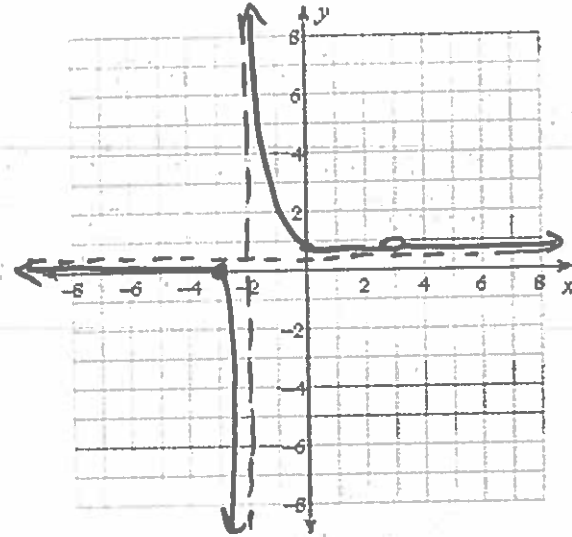
$$x + 2 = 0$$

Hole(s): $x = 3$

End Behavior

Asymptote: $y = 1/2$

$n = 2 \quad m = 2$



$$c) f(x) = \frac{3x - 12}{x^3 - 2x^2 - 8x} = \frac{3(x-4)}{x(x^2 - 2x - 8)} = \frac{3(x-4)}{x(x-4)(x+2)}$$

x-intercept(s): None

y-intercept: None

$$y = \frac{0 - 12}{0 - 0 - 0}$$

Vertical Asymptote(s): $x = 0, x = -2$

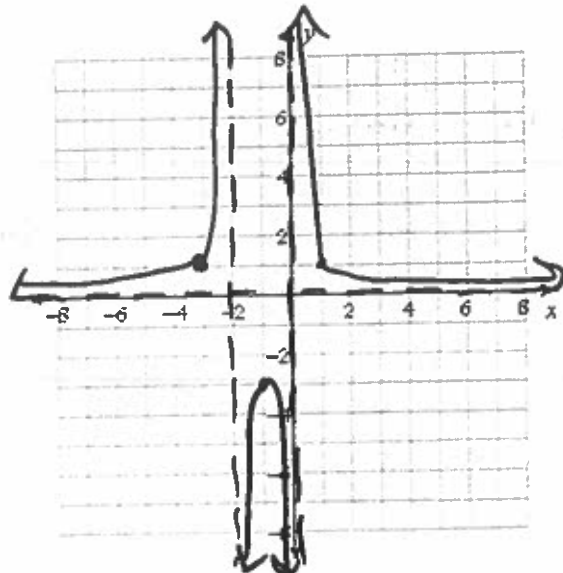
$$x = 0 \quad x + 2 = 0$$

Hole(s): $x = 4$

End Behavior

Asymptote: $y = 0$

$n = 1 \quad m = 3 \quad n < m$



4. Solve each equation. Be sure to check for extraneous solutions.

a) $\frac{1}{6} = \frac{1}{6x} - \frac{x-5}{6x}$

b) $\frac{1}{x} + \frac{7}{x} = \frac{1}{x-5}$

c) $\frac{5x+3}{x^2-6x} + \frac{1}{x-6} = \frac{x+4}{x-6}$

d) $\frac{3}{4x+24} = \frac{3}{2x} - \frac{5x-5}{4x+24}$

5. The number of squirrels at any time t (in years) in a rural area is given by: $P(t) = \frac{750+100t}{15+1.5t}$

a) Find the population of squirrels when the value of t is 10, 50, and 150.

$$P(10) = \frac{750+100(10)}{15+1.5(10)} = 58.3 \approx 58/59$$

$$P(50) = \frac{750+100(50)}{15+1.5(50)} = 63.9 \approx 63/64$$

$$P(150) = \frac{750+100(150)}{15+1.5(150)} = 65.6 \approx 65/66$$

b) What is the x-intercept? What is the significance of this number?

$$\begin{aligned} 750+100t &= 0 \\ 100t &= -750 \end{aligned}$$

$t = -7.5$ At -7.5 years, there are zero squirrels

c) Use the given coordinate system to sketch the function. (Find key features before you graph)

$$\begin{aligned} \text{X-int: } -7.5 & \quad \text{V.A. } 15+1.5t=0 \\ & \quad 1.5t=-15 \\ & \quad t=-10 \end{aligned}$$

$$\begin{aligned} \text{E.B. } y &= \frac{100}{1.5} = 66.7 \\ n &= 1 \quad m = 1 \end{aligned}$$

