HPC/RPC Review
Rational Functions

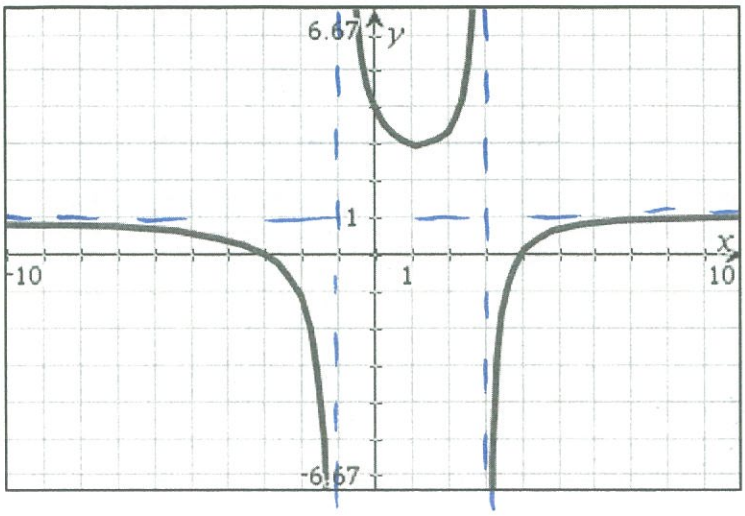
Name key

Date _____ Period _____

PC Reporting Strand: Functions (Identify key features of graphs) Score _____

1. Given the graph of rational functions below, identify all key features of each.

a)



x-intercept(s): (-3,0) (4,0)

y-intercept: (0,4)

Vertical Asymptote(s): x=-1, x=3

End Behavior Asymptote: y = 1

2. Given the equations below identify all key features of the rational function, then graph it.

a) $f(x) = \frac{3}{x+4}$

x-intercept(s) None

$3 \neq 0$

y-intercept $(0, \frac{3}{4})$

$f(0) = \frac{3}{0+4}$

Holes None

Vertical Asymptote(s) $x = -4$

$x+4=0$

$x=-4$

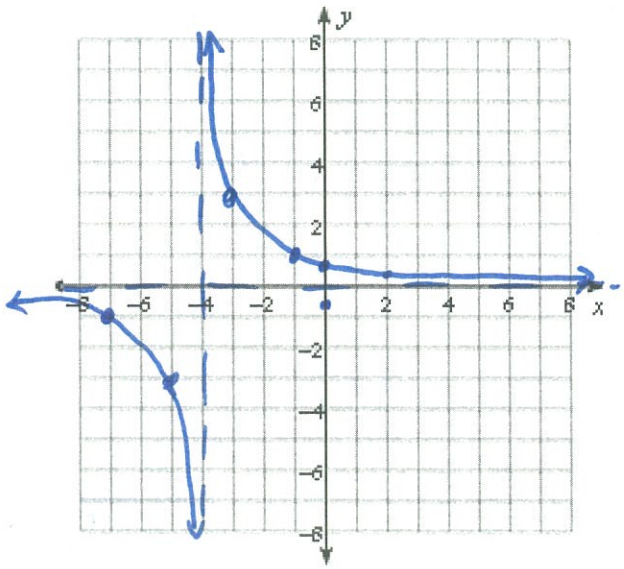
End Behavior $y = 0$

$n=0$

$m=1$

$n<m$

x	y
-7	-1
-5	-3
-3	3
-1	1
2	0.5



b) $f(x) = \frac{-4x-4}{x^2+x-2} = \frac{-4(x+1)}{(x+2)(x-1)}$

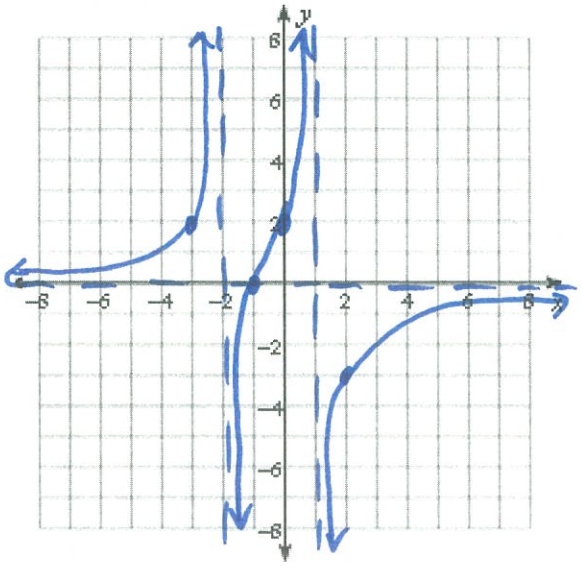
x-intercept(s) $(-1, 0)$ y-intercept $(0, 2)$ Holes None

$-4x-4=0$
$-4x=4$
$x=-1$

$f(0) = \frac{0-4}{0+0-2} = \frac{-4}{-2}$

Vertical Asymptote(s) $x=-2$, $x=1$

$x+2=0$ $x-1=0$



End Behavior $y=0$

$n=1$
$m=2$ $n<m$

x	y
-3	2
Test Points -1.99	$\frac{+}{-} = -$
0.99	$\frac{-}{-} = +$
2	-3

c) $f(x) = -\frac{1}{x-4} + 1 = \frac{-1+(x-4)}{x-4} = \frac{x-5}{x-4}$

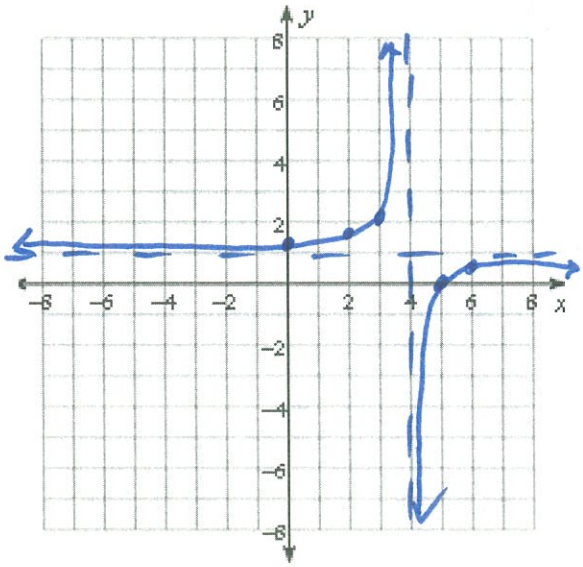
x-intercept(s) $(5, 0)$ y-intercept $(0, 5/4)$ Holes None

$x-5=0$

$f(0) = \frac{0-5}{0-4}$

Vertical Asymptote(s) $x=4$

$x-4=0$



End Behavior $y=1$

$n=1$
$m=1$ $n=m$

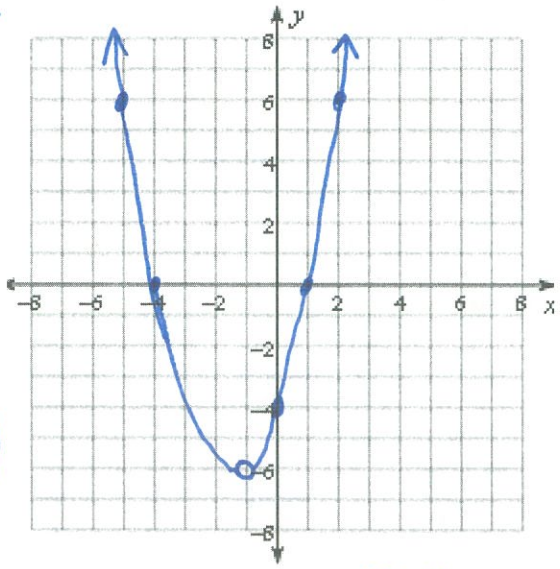
x	y
2	$+3/2 = +1.5$
3	2
6	0.5

HONORS ONLY

d) $f(x) = \frac{x^3+4x^2-x-4}{x+1} = \frac{x^2(x+4)-1(x+4)}{x+1} = \frac{(x^2-1)(x+4)}{(x+1)} = \frac{(x+1)(x-1)(x+4)}{x+1}$

x-intercept(s) $(1,0)$ $(-4,0)$ y-intercept $(0,-4)$ Holes $x+1=0 \quad x=-1$

$x-1=0 \quad x+4=0$ $f(0) = \frac{0+0-0-4}{0+1} = \frac{-4}{1}$



Vertical Asymptote(s) None

End Behavior $y = x^2+3x-4$

$n=3$

$m=1$

$n>m$... but after factoring it would be $\frac{x^2+3x-4}{1}$ so... not really an asymptote

$-1 \mid 1 \quad 4 \quad -1 \quad -4$

$\quad\quad\downarrow \quad -1 \quad -3 \quad 4$

$\quad\quad 1 \quad 3 \quad -4 \quad 0$

x	y
-5	6
-1	-6 ← hole
2	6

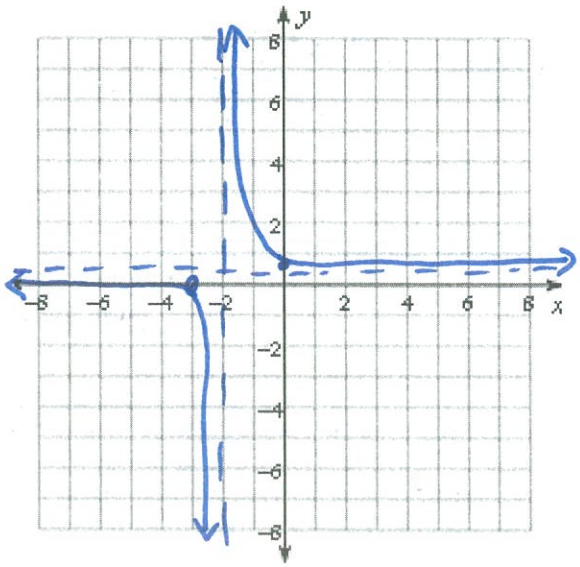
e) $f(x) = \frac{x^2-9}{2x^2-2x-12} = \frac{(x+3)(x-3)}{2(x^2-x-6)} = \frac{(x+3)(x-3)}{2(x+2)(x-3)}$

x-intercept(s) $(-3,0)$ y-intercept $(0, \frac{3}{4})$ Holes $x-3=0 \quad x=3$

$x+3=0$

$x=-3$

$f(0) = \frac{0-9}{0-0-12} = \frac{-9}{-12} = \frac{3}{4}$



Vertical Asymptote(s) $x=-2$

$x+2=0$

End Behavior $y = \frac{1}{2}$

$n=2$

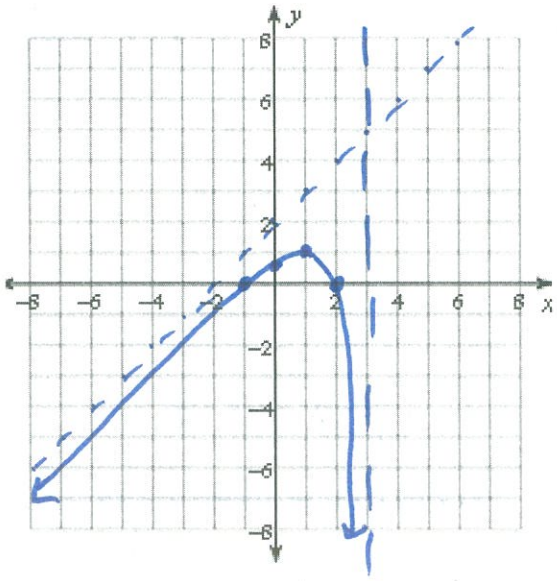
$m=2$ $n=m$

x	y

f) $f(x) = \frac{x^2-2x+3}{x+2}$ $f(x) = \frac{x^2-x-2}{x-3} = \frac{(x+1)(x-2)}{x-3}$

x-intercept(s) $(-1,0)\ (2,0)$ y-intercept $(0, \frac{2}{3})$ Holes None

$x+1=0 \quad x-2=0$ $f(0) = \frac{0-0-2}{0-3} = \frac{-2}{-3}$



Vertical Asymptote(s) $x=3$

$x-3=0$

End Behavior $y = x+2$

$n=2$ $m=1$ $n>m$

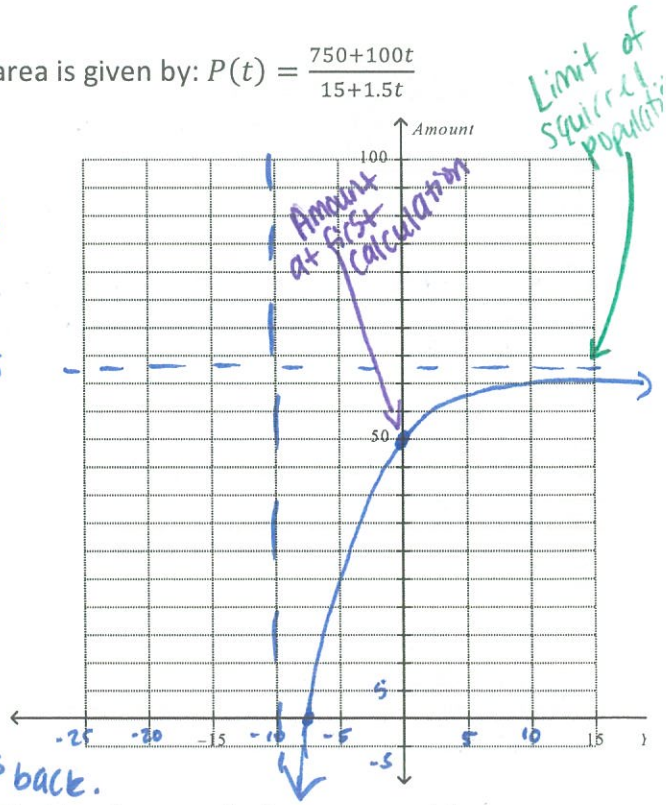
$3 \mid 1 \quad -1 \quad -2$
$\quad\quad\ \ 3 \quad\ 6$
$\quad 1 \quad\ 2 \quad\ 4$

5. The number of squirrels at any time t (in years) in a rural area is given by: $P(t) = \frac{750+100t}{15+1.5t}$

a) Find the population of squirrels when the value of t is:

10 → $t(10) = \frac{750+100(10)}{15+1.5(10)} = 58.3$ → about 58 squirrels

50 → $t(50) = \frac{750+100(50)}{15+1.5(50)} = 63.8$ → 63 squirrels



b) What is the t-intercept? $750+100t=0$ $100t=-750$ $t=-7.5$

What is the significance of this number?

In reality nothing because "t" can't be negative... but maybe they have data 7.5 years back.

c) Use the given coordinate system to sketch the function. (Find key features before you graph)

$t(x\text{-int}) = (-7.5, 0)$

Vertical: $15+1.5t=0$ $1.5t=-15$ $t=-10$

End Behavior: $n=1$ $m=1$ $y = \frac{100}{1.5} = 66.\overline{6}$

$y\text{-int} = P(0) = \frac{750+0}{15+0} = 50 \quad (0,50)$