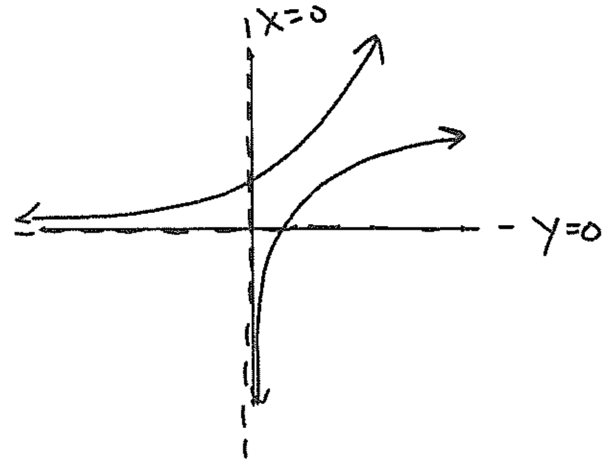
Logarithmic Functions

A Logarithmic Function is the inverse of an exponential function.



Changing between logarithmic and exponential form

If $x > 0$, $b > 0$ and $b \neq 1$,
then $log_b(x) = y$ if and only if $b^y = x$

Example 1: Evaluating Logarithms by rewriting the logarithmic expression

a. $\log_2 4 = x$

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

$2^x = 4$

$x = 2$

b. $\log_9 1 = y$

$9^y = 1$ $y = 0$

c. $\log_5 \sqrt{5} = z$

$5^z = \sqrt{5}$

$5^z = 5^{1/2}$

$z = \frac{1}{2}$

d. $\log_6 6 = a$

$6^a = 6^1$

$a = 1$

e. $\log_3 0 = m$

$3 = 0$ UNDEFINED

f. $\log_7 (-49) = z$ UNDEF.

$7^z = -49$

$f(f^{-1}(x)) = x$

$\log_{18} 18^{34} = 34 \rightarrow$

Basic Properties of Logarithms

If $x > 0$, $b > 0$ and $b \neq 1$ and any real number y:

- $\log_b 1 = 0$ because $b^0 = 1$
- $\log_b b = 1$ because $b^1 = b$
- $\log_b b^y = y$ because $b^y = b^y$
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$ $\rightarrow b^{(\log_b x)} = x$

Example 2: Evaluating Logarithmic Expressions by using logarithmic properties

a. $\log_2 8 =$ $\log_2 2^3 = 3$

b. $\log_3 \sqrt{3} =$ $\cancel{\log_3 3^{1/2}} = \frac{1}{2}$

c. $\cancel{6^{\log_6 11}} =$ 11

Common Logarithms--Base 10

$x = 1x^1$

$\log_{10} x = \log x$

Common logarithms are logarithms with base 10

$y = \log x$ if and only if $10^y = x$

Basic Properties of Common Logarithms

If $x > 0$, $b > 0$ and $b \neq 1$ and any real number y:

- $\log 1 = 0$ because $10^0 = 1$
- $\log 10 = 1$ because $10^1 = 10$
- $\log 10^y = y$ because $10^y = 10^y$
- $10^{\log x} = x$ because $\log x = \log x$

Example 3: Evaluating Base 10 Logarithmic Expressions

a. $\log 100 =$ $\log 10^2 = 2$

b. $\log \sqrt{10} =$ $\frac{1}{2}$ $10^{1/2}$

c. $10^{\log 6} =$ 6

Example 4: Solving Simple Logarithmic Equations

a. $\log x = 5$

$10^5 = x$

$100{,}000 = x$:)

b. $\log_x 81 = 4$

$x^4 = 81$

$x = 3$

c. $\log_6 x = 3$

$6^3 = x$

$216 = x$

Common Logarithms--Base e

$\ln = \log_e$

Natural logarithms are logarithms with base e

Pe^{rt}

$y = \ln x$ if and only if $e^y = x$

Basic Properties of Natural Logarithms

If $x > 0$, $b > 0$ and $b \neq 1$ and any real number y:

- $\ln 1 = 0$ because $e^0 = 1$
- $\ln e = 1$ because $e^1 = e$
- $\ln e^y = y$ because $e^y = e^y$
- $e^{\ln x} = x$ because $\ln x = \ln x$

Example 5: Evaluating Base e Logarithmic Expressions

a. $\ln e^5 = 5$ b. $\ln \sqrt{e} = \frac{1}{2}$ c. $e^{\ln 4} = 4$

$$\ln \sqrt[7]{e^2} = \frac{2}{7}$$

$\log_e x = 1$

$e^1 = x$

Logarithmic Functions: Intro Name Key

Rewrite each logarithmic expression in exponential form.

1. $\log_2 4 = 2$
$2^2 = 4$

2. $\log_3 27 = 3$
$3^3 = 27$

3. $\log_{10} 10{,}000 = 4$
$10^4 = 10{,}000$

4. $\log_6 1 = 0$
$6^0 = 1$

5. $\ln e^2 = 2$
$2 = 2$

6. $\log_n k = w$
$n^w = k$

Rewrite each exponential expression in logarithmic form.

7. $4^2 = 16$
$\log_4 16 = 2$

8. $3^5 = 243$
$\log_3 243 = 5$

9. $36^{1/2} = 6$
$\log_{36} 6 = \frac{1}{2}$

10. $10^6 = 1{,}000{,}000$
$\log_{10} 1{,}000{,}000 = 6$

11. $923^0 = 1$
$\log_{923} 1 = 0$

12. $T^S = A$
$\log_T A = S$

Solve each logarithmic equation (for x) without using a calculator.

13. $\log_5 25 = x$
$5^x = 25$
$x = 2$

14. $\log_2 32 = x$
$2^x = 32$
$x = 5$

15. $\log_x 36 = 2$
$x^2 = 36$
$x = 6$

16. $\log_3 x = 2$
$3^2 = x$
$9 = x$

17. $\log x = 1000$
$10^{1000} = x$

18. $\ln x = 1$
$e^1 = x$
$e = x$

19. $\ln e^x = 4$
$x = 4$

20. $\log_4 4^x = 9$
$x = 9$ $4^9 = 4^x$

21. $\log_7 x^3 = 3$
$7^3 = x^3$
$x = 7$