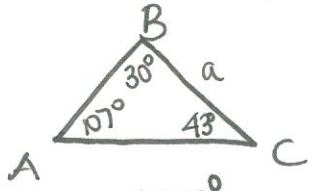
RPC/HPC PreCalculus
Laws of Sines and Cosines Review Problems

Name Key

Level 2

1. Solve the triangles. If two triangles exist, solve for all parts of both triangles.

a. $\angle B = 30°$, $\angle C = 43°$ and $a = 8$ ASA → LOS



$m\angle A = 107°$

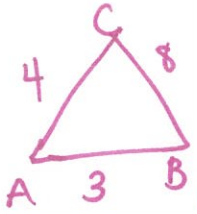
$\frac{\sin 107°}{8} = \frac{\sin 30°}{b}$

$b = \frac{8 \cdot \sin 30°}{\sin 107°} = 4.18$

$\frac{\sin 107°}{8} = \frac{\sin 43°}{c}$

$c = \frac{8 \cdot \sin 43°}{\sin 107°} = 5.71$

b. $a = 8$, $b = 4$, $c = 3$ SSS → LOC



$a^2 = b^2 + c^2 - 2bc\cos A$

$64 = 16 + 9 - 24\cos A$

$64 = 25 - 24\cos A$

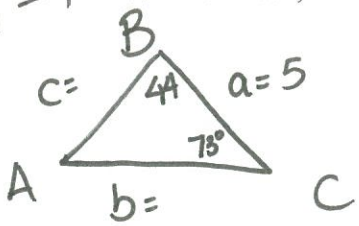
$39 = -24\cos A$

$-1.6250 = \cos A$

$\cos A$ must be between -1 to $+1$

also $3+4 < 7$ so can't form a triangle

Kill → c. $\angle C = 73°$, $\angle B = 44°$, $a = 50$ ~~ASA~~ → LOS



$m\angle A = 180 - (73+44)$

$m\angle A = 63°$

$\frac{\sin 63°}{5} = \frac{\sin 44°}{b}$

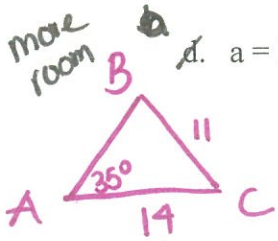
$b = \frac{5\sin 44°}{\sin 63°} = 3.90$

$\frac{\sin 63°}{5} = \frac{\sin 75°}{c}$

$c = \frac{5 \cdot \sin 75°}{\sin 63°} = 5.42$

more room

d. $a = 11$, $b = 14$ and $\angle A = 35°$ SSA – LOS



S2017 2 solns

$\frac{\sin 35°}{11} = \frac{\sin B}{14}$

$\sin B = \frac{14 \cdot \sin 35°}{11} = 0.7300$

$m\angle B = \sin^{-1}(0.7300) = 46.9°$

$m\angle C = 180° - (35+46.9) = 98.1°$

$m\angle B_S = 133.1°$

$\angle C_2 = 12°$

$\frac{\sin 35°}{11} = \frac{\sin 12°}{c}$

$c = \frac{11\sin 12°}{\sin 35°}$

$c_2 = 3.987$

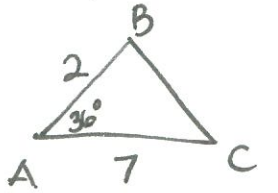
11

4

$\frac{\sin 98.1°}{c} = \frac{\sin 35°}{11}$

$c = \frac{11 \cdot \sin 98.1°}{\sin 35°} = 18.99$

e. c = 2, b = 7 and ∠A = 36° SAS – LOC



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 7^2 + 2^2 - 2(2)(7)\cos 36°$

$a^2 = 49 + 4 - 28\cos 36°$

$a^2 = 53 - 22.65$

$a^2 = 30.35$ $a \approx 5.5$

$\frac{\sin 36°}{5.5} = \frac{\sin C}{2}$

$\sin C = \frac{2 \cdot \sin 36°}{5.5}$

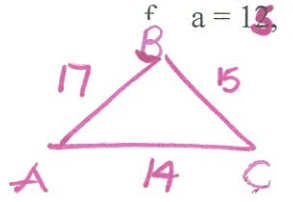
$\sin C = 0.2137$

$m\angle C = 12.34$

$m\angle B = 180 - (36 + 12.3)$

$m\angle B = 131.7°$

f. a = 13, b = 14, c = 17 SSS – LOC



$c^2 = a^2 + b^2 - 2ab\cos C$

$17^2 = 15^2 + 14^2 - 2(15)(14)\cos C$

$289 = 225 + 196 - 420\cos C$

$-132 = -420\cos C$

$\cos C = 0.3143$ $m\angle C = 71.7°$

$\frac{\sin 71.7}{17} = \frac{\sin B}{14}$

$\sin B = \frac{14 \cdot \sin 71.7°}{17}$

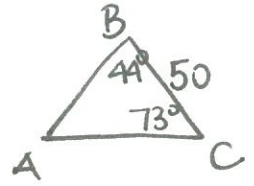
$\sin B = 0.7819$

$m\angle B = 51.4$

$m\angle A = 180° - (71.7 + 51.4)$

$m\angle A = 56.9°$

g. m∠C = 73°, m∠B = 44°, CB = 50 ASA – LOS



$m\angle A = 180 - (44 + 73°)$

$m\angle A = 63°$

$\frac{\sin 63°}{50} = \frac{\sin 44}{b}$

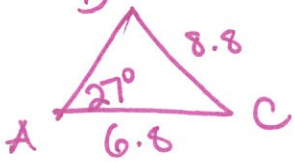
$b = \frac{50 \cdot \sin 44°}{\sin 63°}$

$b = 39.0$

$\frac{\sin 63°}{50} = \frac{\sin 73°}{C}$

$C = \frac{50 \cdot \sin 73°}{\sin 63°} = 53.7$

h. a = 8.8, b = 6.8 and A = 27° SSA – LOS



$\frac{\sin 27°}{8.8} = \frac{\sin B}{6.8}$

$\sin B = \frac{6.8(\sin 27°)}{8.8}$

$\sin B = 0.3508$

$m\angle B = 20.5°$

$m\angle B_S = 159.5°$

$m\angle B_S + m\angle A$

$159.5 + 27 = 184.5 > 180°$ so 1 soln

$m\angle C = 180 - (27 + 20.5) = 132.5°$

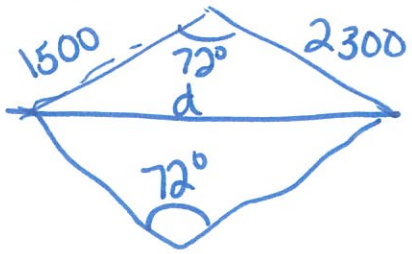
$\frac{\sin 132.5°}{C} = \frac{\sin 27°}{8.8}$

$C = \frac{(8.8)\sin 132.5°}{\sin 27°}$

$C = 14.3$

Level 3

2. An engineer needs to design a bridge for a road that must cross a canyon. From a helicopter, he measures the angle between the two sides of the canyon to be 72°. If the helicopter is 1500 feet from one edge of the canyon and 2300 feet from the other edge, how long must the bridge be to span the canyon?



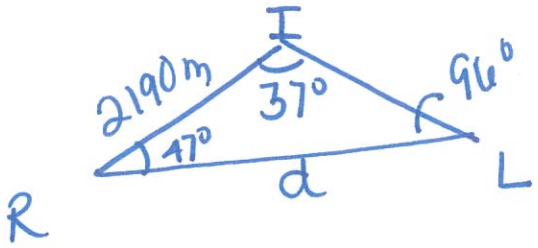
SAS

$d^2 = 1500^2 + 2300^2 - 2(1500)(2300)\cos 72°$

$d^2 = 5497783$

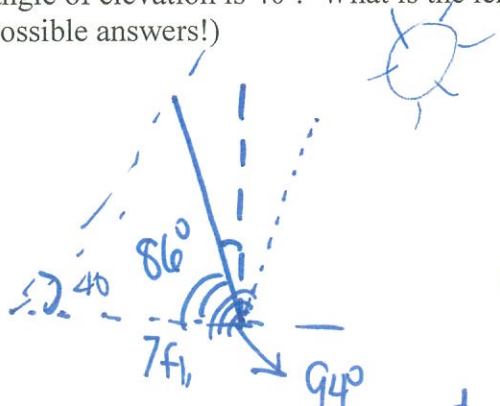
$d \approx 2325.5$

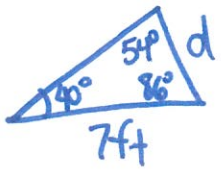
3. A ship at sea, the Intrepid, spots two other ships, the Ranger and the Lancer, and measures the angle between to be 37°. The distance between the Intrepid and the Ranger is 2190 meters. The Ranger measures an angle of 47° between the Intrepid and the Lancer. To the nearest meter, what is the distance between the Ranger and the Lancer

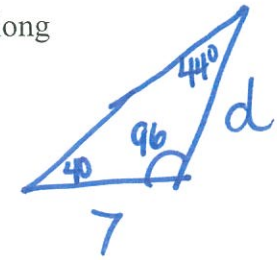


$\frac{\sin 96°}{2190} = \frac{\sin 37°}{d} \rightarrow d = \frac{2190 \sin 37°}{\sin 96°} = 1325.2\text{m}$

4. A street sign is leaning at a 4° angle from the vertical. It casts a shadow that is 7 feet long when the sun's angle of elevation is 40°. What is the length of the street sign's post? (There are two possible answers!)





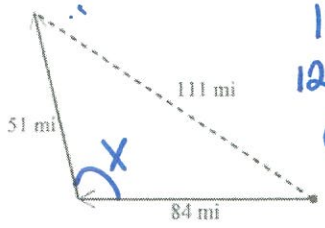


$\frac{\sin 40}{d} = \frac{\sin 54°}{7}$ or $\frac{\sin 40}{d} = \frac{\sin 44}{7}$

$d = \frac{7 \cdot \sin 40}{\sin 54}$ or $d = \frac{7 \sin 40°}{\sin 44°}$

$d = 5.56\text{ ft}$ $d = 6.48\text{ ft}$

5. A ship travels due west for 84 miles. It then travels in a northern direction for 51 miles and ends up 111 miles from its original position. How many degrees did it turn when it changes direction? Round your answer to the nearest tenth.



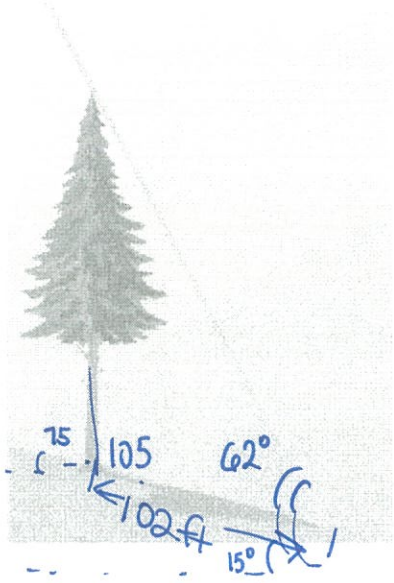
$111^2 = 84^2 + 51^2 - 2(51)(84)\cos X$

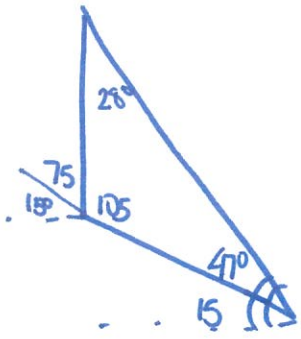
$12321 = 7056 + 9057 - 8568\cos A$

$\cos A = 0.5124$

$m\angle A \approx 59.2°$

6. A tree growing on a hillside casts a 102 foot shadow straight down the hill. Find the vertical height of the tree if, relative to the horizontal, the hill slopes 15° and the angle of elevation of the sun is 62°





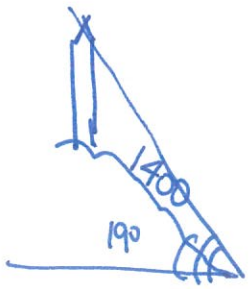
$\frac{\sin 28°}{107} = \frac{\sin 47°}{h}$

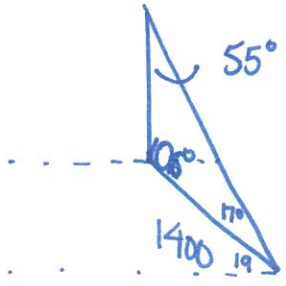
$h = \frac{107 \cdot \sin 47°}{\sin 28°}$

$h \approx 166.7$ ft

finish problem

7. An obelisk stands on a mountain that has a slope of 19° from the horizontal. From a point 1400 feet down the mountain the angle of elevation to the top of the ~~tree~~ obelisk is 36°. How tall is the





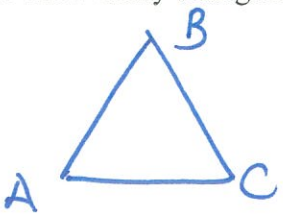
$\frac{\sin 55°}{1400} = \frac{\sin 17°}{O}$

$O = \frac{\sin 17° (1400)}{\sin 55°}$

$O = 499$ ft tall

Level 4

8. How many triangles exist if: a = 42.2, b = 37 and A = 112°?



$h = b \sin 112°$

$h =$

Obtuse
$a > b$ one triangle

9. How many triangles exist if: b = 9.3, c = 41 and B = 18°?

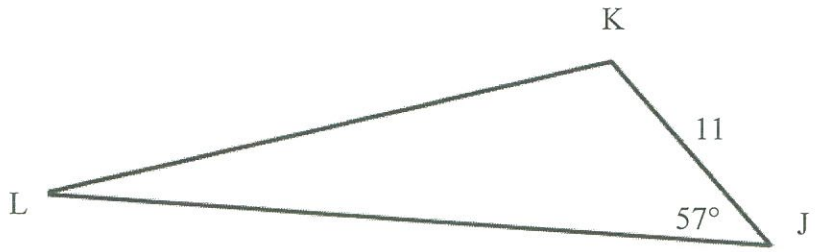
$h = c \sin B$
$h = 41 \sin 18° = 12.67$
$b < h$ so no triangle

10. How many triangles exist if: a = 95, c = 125 and A = 49°?

$h = c \sin A$
$h = 125 \sin 49° = 94.3$

$h < a < c$
$94.3 < 95 < 125$ two triangles

11. For the given triangle JKL, determine a value for KL that would create the given number of triangles. Explain your reasoning for parts a through c below



a. One Triangle $f > 9.23$

b. Two Triangles $9.23 < f < 11$

c. No Triangles $f < 9.23$

Explanation

$h = 11 \sin 57° = 9.23$