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# Honors PreCalculus

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Series

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Give it a try:

Find the sum of the natural numbers  
1 to 100. Without a calculator 😊

$$1 + 2 + 3 + \dots + 99 + 100 = ?$$

**5050**

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# Summation Notation

- The sum of the terms of a sequence  $a_k = \{a_1, a_2, a_3, \dots, a_n\}$  can be represented as

A hand-drawn diagram of the summation notation  $\sum_{k=1}^n a_k$ . The number  $n$  at the top is circled in blue, with a red arrow pointing to it from the word "sum" written in red. The summation symbol  $\Sigma$  and the term  $a_k$  are circled in red. The number  $k=1$  at the bottom is circled in blue, with the word "start" written in blue below it.

“The sum of  $a_k$  from  $k = 1$  to  $k = n$ ”

# Summation Notation

- Find the sums:

$$\sum_{k=1}^5 3k = 3(1) + 3(2) + 3(3) + 3(4) + 3(5) = 45$$

*Handwritten notes: A red circle around the upper limit 5 with an arrow pointing to the summand. A red circle around the lower limit 1 with an arrow pointing to the summand. Red text below the summands: 3(5) + 3(4) - - -*

$$\sum_{k=5}^8 k^2 = 5^2 + 6^2 + 7^2 + 8^2 = 174$$

$$\sum_{n=0}^4 \cos n\pi = \cos 0 + \cos \pi + \cos 2\pi + \cos 3\pi + \cos 4\pi = 1$$

$$\begin{aligned} \sum_{x=1}^{\infty} \frac{3}{10^x} &= \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \dots \\ &= 0.\underline{3} + 0.0\underline{3} + 0.00\underline{3} + \dots \\ &= 0.33\overline{3} = \frac{1}{3} \end{aligned}$$

# Sum of a finite arithmetic sequence

- If  $a_k = \{a_1, a_2, a_3, \dots, a_n\}$ , then

$$\sum_{k=1}^n a_k = a_1 + a_2 + \dots + a_n$$

$$= n \left( \frac{a_1 + a_n}{2} \right)$$

\*Reminder\*  
 $a_n = a_1 + (n-1)d$   
sequence

Where:  $a_1$  is the initial term

$a_n$  is the final term

$n$  is the number of terms

$d$  is the common difference

# Sum of a finite arithmetic sequence

- Example: A lecture hall has 8 seats in the first row and 24 in the last row. If each row has 2 more seats than the one in front of it, how many total seats are in the room?

$$a_1 = 8 \quad a_n = 24 \quad d = 2 \quad n = ????????$$

$$\Sigma = \frac{n}{2} (a_1 + a_n)$$

*Handwritten notes: A red circle highlights the formula. Blue arrows point from the number 8 to  $a_1$  and from the number 24 to  $a_n$ . A blue arrow points from the number 9 to the  $n$  in the numerator.*

$$\text{find } n: a_n = a_1 + (n - 1)d$$

$$24 = 8 + (n - 1)(2)$$

$$24 = 8 + 2n - 2$$

$$24 = 2n + 6$$

$$18 = 2n$$

$$9 = n$$

$$\sum_{k=1}^n a_k = 9 \left( \frac{8 + 24}{2} \right)$$

$$= 9 \left( \frac{32}{2} \right)$$

$$= 9(16)$$

$$= 144$$

# Sum of a finite geometric sequence

- If  $a_k = \{a_1, a_2, a_3, \dots, a_n\}$ , then

$$\sum_{k=1}^n a_k = a_1 + a_2 + \dots + a_n$$

$$= \frac{a_1(1-r^n)}{1-r}$$

\* Reminder \*

$$a_n = a_1(r^{n-1})$$

Where:  $a_1$  is the initial term

$n$  is the number of terms

$r$  is the common ratio

# Sum of a **finite** geometric sequence

- Example: Find the sum of:  $4, -\frac{4}{3}, \frac{4}{9}, -\frac{4}{27}, \dots, 4\left(-\frac{1}{3}\right)^{10}$

$a_1 = 4$     $r = -\frac{1}{3}$     $n^{\text{th}}$  term is  $4\left(-\frac{1}{3}\right)^{10}$ , so  $n = 11$

$\frac{a_1(1-r^n)}{1-r}$

$$\sum_{k=1}^{11} a_k = \frac{4\left(1 - \left(-\frac{1}{3}\right)^{11}\right)}{1 - \left(-\frac{1}{3}\right)} = \frac{4\left(1 - \left(-\frac{1}{3}\right)^{11}\right)}{\frac{4}{3}} = \frac{1 - \left(-\frac{1}{3}\right)^{11}}{\frac{1}{3}}$$

$$= 3\left(1 - \left(-\frac{1}{3}\right)^{11}\right) \approx 3.000016935$$



# Sum of an infinite geometric sequence

- An **infinite** geometric series **converges** if  $|r| < 1$

If this is the case, then the sum of the series is:

$$\sum_{k=1}^{\infty} a_k r^{k-1} = \frac{a_1}{1-r}$$

Where:  $a_1$  is the initial term

$r$  is the common ratio

# Sum of an infinite geometric sequence

$$a_n = a_1 (r^{n-1})$$

- Find the sums (if the series converges):

1.  $\sum_{k=1}^{\infty} 3(0.75)^{n-1}$   $|r|=0.75 < 1$ , so it converges  $a_1 = 3$ , so

$$\sum_{k=1}^{\infty} 3(0.75)^{n-1} = \frac{a_1}{1-r} = \frac{3}{1-0.75} = \frac{3}{0.25} = 12$$

$$\frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$$

2.  $\sum_{x=1}^{\infty} \frac{3}{10^x}$

$$a_1 = \frac{3}{10}$$

$$r = \frac{1}{10}$$

$$= \frac{\frac{3}{10}}{1 - \frac{1}{10}} = \frac{\frac{3}{10}}{\frac{9}{10}} = \frac{3}{10} \cdot \frac{10}{9} = \frac{3}{9} = \frac{1}{3}$$