Honors PreCalculus

Series

Give it a try:

Find the sum of the natural numbers 1 to 100. Without a calculator ☺

$$1 + 2 + 3 + \ldots + 99 + 100 = ?$$

5050

Summation Notation

- The sum of the terms of a sequence
  $a_k = \{a_1, a_2, a_3, \ldots a_n\}$ can be represented as

$$\sum_{k=1}^{n} a_k$$

sum

start

"The sum of a_k from $k = 1$ to $k = n$"

Summation Notation

- Find the sums:

$$\sum_{k=1}^{5} 3k = 3(1)+3(2)+3(3)+3(4)+3(5) = 45$$

$3(5)+3(4)$ . . .

$$\sum_{k=5}^{8} k^2 = 5^2+6^2+7^2+8^2 = 174$$

$$\sum_{n=0}^{4} \cos n\pi = \cos 0 + \cos \pi + \cos 2\pi + \cos 3\pi + \cos 4\pi = 1$$

$$\sum_{x=1}^{\infty} \frac{3}{10^x} = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \cdots$$

$$= 0.3 + 0.03 + 0.003 + \cdots$$

$$= 0.33\overline{3} = \frac{1}{3}$$

Sum of a finite arithmetic sequence

Reminder
$a_n = a_1 + (n-1)d$
sequence

- If $a_k = \{a_1, a_2, a_3, \ldots, a_n\}$, then

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$$

$$\boxed{= n\left(\frac{a_1 + a_n}{2}\right)}$$

Where: a_1 is the initial term

a_n is the final term

n is the number of terms

d is the common difference

Sum of a finite arithmetic sequence

- Example: A lecture hall has 8 seats in the first row and 24 in the last row. If each row has 2 more seats than the one in front of it, how many total seats are in the room?

$$\Sigma = \overset{9}{n}\left(\frac{\overset{8}{a_1} + \overset{24}{a_n}}{2}\right)$$

$a_1 = 8$ $a_n = 24$ $d = 2$ $n = ???????$

find n: $a_n = a_1 + (n - 1)d$

$$24 = 8 + (n - 1)(2)$$

$$24 = 8 + 2n - 2$$

$$24 = 2n + 6$$

$$18 = 2n$$

$$9 = n$$

$$\sum_{k=1}^{n} a_k = 9\left(\frac{8+24}{2}\right)$$

$$= 9\left(\frac{32}{2}\right)$$

$$= 9(16)$$

$$= 144$$

Sum of a finite geometric sequence

- If $a_k = \{a_1, a_2, a_3, \ldots, a_n\}$, then

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$$

$$= \frac{a_1\left(1 - r^n\right)}{1 - r}$$

* Reminder *

$a_n = a_1 (r^{n-1})$

Where: a_1 is the initial term

n is the number of terms

r is the common ratio

Sum of a **finite** geometric sequence

- Example: Find the sum of: $4, -\frac{4}{3}, \frac{4}{9}, -\frac{4}{27}, \cdots, 4\left(-\frac{1}{3}\right)^{10}$

$4\left(\frac{-1}{3}\right)^0$

$a_1 = 4 \quad r = -\frac{1}{3} \qquad n^{th}$ term is $4\left(-\frac{1}{3}\right)^{10}$, so n = 11

$$\frac{a_1(1-r^n)}{1-r}$$

$$\sum_{k=1}^{11} a_k = \frac{4\left(1-\left(-\frac{1}{3}\right)^{11}\right)}{1-\left(-\frac{1}{3}\right)} = \frac{4\left(1-\left(-\frac{1}{3}\right)^{11}\right)}{\frac{4}{3}} = \frac{1-\left(-\frac{1}{3}\right)^{11}}{\frac{1}{3}}$$

$$= 3\left(1-\left(-\frac{1}{3}\right)^{11}\right) \boxed{\approx 3.000016935}$$

Sum of an infinite geometric sequence

- An infinite geometric series converges **if |r|<1**

  If this is the case, then the sum of the series is:

$$\sum_{k=1}^{\infty} a_k r^{k-1} = \frac{a_1}{1-r}$$

Where: a_1 is the initial term

r is the common ratio

Sum of an infinite geometric sequence

$a_n = a_1 (r^{n-1})$

- Find the sums (if the series converges):

1. $\sum_{k=1}^{\infty} 3(0.75)^{n-1}$ |r|=0.75<1, so it converges a_1 = 3, so

$$\sum_{k=1}^{\infty} 3(0.75)^{n-1} = \frac{a_1}{1-r} = \frac{3}{1-0.75} = \frac{3}{0.25} = 12$$

$\frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$

2. $\sum_{x=1}^{\infty} \frac{3}{10^x}$ $= \frac{\frac{3}{10}}{1-\frac{1}{10}} = \frac{\frac{3}{10}}{\frac{9}{10}} = \frac{3}{\cancel{10}} \cdot \frac{\cancel{10}}{9} = \frac{3}{9} = \boxed{\frac{1}{3}}$

$a_1 = \frac{3}{10}$

$r = \frac{1}{10}$