Cornell Notes	Topic/Objective:	Name:
Inverse Functions	*Students will determine if a function has an inverse & then find the inverse.*	Class/Period:
		Date:

Essential Question: How do you determine if a function has an inverse?

Standard: Build new functions from existing functions (F-BF.4a - Inverse Functions)

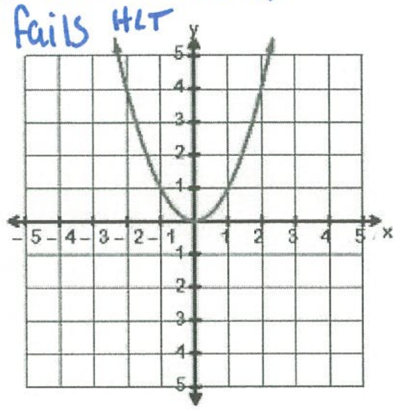
Questions:

Notes:

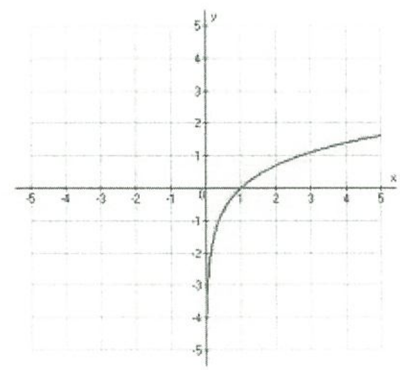
Horizontal Line Test (HLT): If a horizontal line passes through a function 2 or more times, then the function is non-invertible.

Example 1: Determine if the following functions have an inverse using the HLT.

a. Non-invertible, fails HLT



b. Invertible



If a function passes the HLT, then inverse can be algebraically by following these steps:

1. Replace $f(x)$ with y.
2. Switch x & y.
3. Solve for y.
4. Replace y with $f^{-1}(x)$.

Summary:

Questions:	Notes:

Example 1: Find an equation for $f^{-1}(x)$ if $f(x)=\dfrac{2x}{2x-1}$

$y = \dfrac{2x}{2x-1}$

$x = \dfrac{2y}{2y-1}$

$x(2y-1) = 2y$

$2xy - x = 2y$

$2xy - 2y = x$

$y(2x-2) = x$

$y = \dfrac{x}{2x-2}$

$\boxed{f^{-1}(x) = \dfrac{x}{2x-2}}$

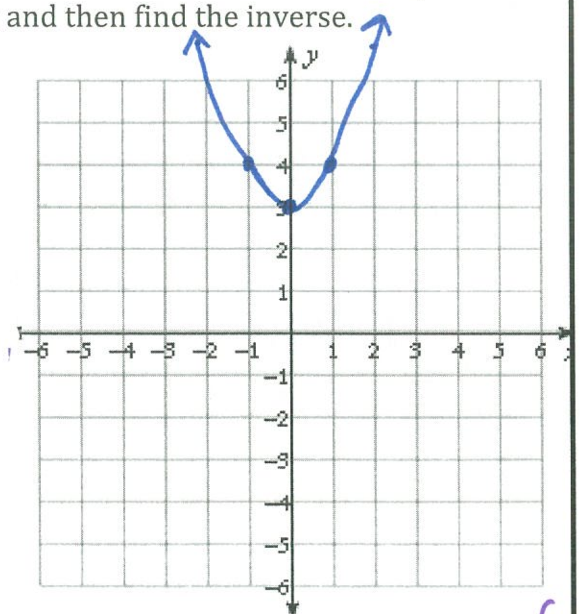
If a function fails the HLT, then the domain needs to be restricted to produce an inverse.

Example 2: Graph $f(x) = x^2 + 3$. Determine if it is invertible. If it is, find the inverse. If it is not, restrict the domain and then find the inverse.

Fails HLT, non-invertible

*To restrict the domain, choose where it passes HLT

$[0, \infty)$



$y = x^2 + 3$

$x = y^2 + 3$

$x - 3 = y^2$

$\pm\sqrt{x-3} = y$

$f^{-1}(x) = \sqrt{x-3}$ Domain of $f^{-1}(x)$: $[3, \infty)$ *Range of $f(x)$*

Summary:

Questions:	Notes:
How can I tell if functions are inverses of each other?	Two Methods: 4 steps or Composition

Example 3: Show that each function is the inverse of the other.

$f(x) = -2x + 1$ $\qquad$ $g(x) = \frac{-x+1}{2}$

method 1:

$y = -2x + 1$

$x = -2y + 1$

$x - 1 = -2y$

$\frac{x-1}{-2} = y$

$\frac{-x+1}{2} = y$

match ⇒ inverse

method 2:

$f(g(x)) = -2\left(\frac{-x+1}{2}\right) + 1$

$= \frac{2x-2}{2} + 1$

$= x - 1 + 1$

$= x$

$g(f(x)) = \frac{-(-2x+1)+1}{2}$

$= \frac{2x-1+1}{2}$

$= x$

Both simplify to x

⇒ inverses

Summary:

Questions:	Notes:

Example 4: Evaluate the following using the information given.

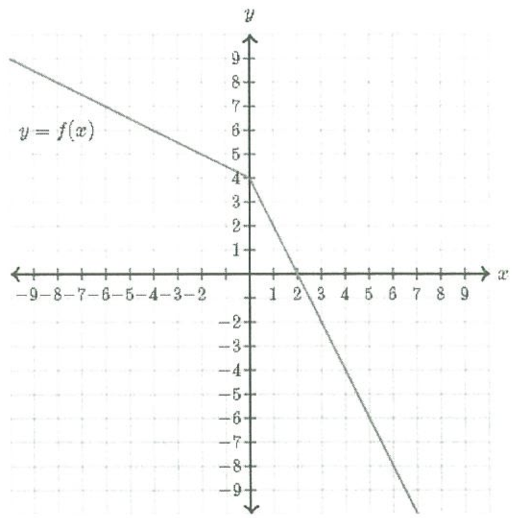
a) Given the table, find $f^{-1}(3)$.

x	f(x) (y)
4	-1
5	4
9	3

Remember domain & range switch

$f^{-1}(3) = 9$

b) Given the graph, find $f^{-1}(4)$.



$f^{-1}(4) = 0$

Just look at the y=4 position.

c) Given the table, find $f^{-1}(8)$ AND $f^{-1}(f^{-1}(13))$.

x	-7	11	-13	6	5	-9
$f(x)$	7	12	8	-7	13	5

$f^{-1}(8) = -13$ $f^{-1}(f^{-1}(13)) = -9$

Summary: