Law of Cosines

Students will learn to find the sides and angles of acute and obtuse triangles.

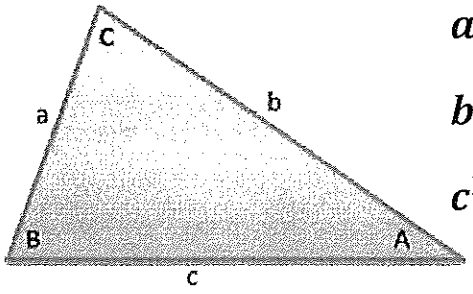
Precalculus

KEY

What is the law of cosines?

Law of Cosines

Let Δ ABC be any triangle with sides and angles labeled as below:



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

There are two cases in which the law of cosines can be used to find the missing sides or angles or a triangle.

How do you find the missing side and angles of a triangle when you know two sides and the included angle?

Case 1: *Side-Angle-Side (SAS)*

1. Use Law of Cosines to find the side opposite the given angle.
2. Use the Law of Sines to find the angle opposite the shorter of the two given sides. This angle is always acute.
3. Find the third angle by using the fact that the sum of the angles of a triangle is 180°.

6.2 Law of Cosines

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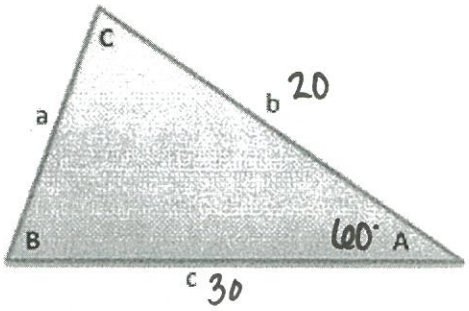
Precalculus

* Round angles to nearest WHOLE

Example 1: ***Use the law of cosines to solve the triangle.***



$m\angle A = 60°$

$b = 20$

$c = 30$

$a = 26.5$

$\angle B = 40.9°$

$\angle C = 79.1°$

$a^2 = 20^2 + 30^2 - 2(20)(30)\cos 60°$

$a = \sqrt{\quad}$

$a \approx 26.5$

$\frac{\sin B}{20} = \frac{\sin 60}{26.5}$

$B = \sin^{-1}\left(\frac{20 \cdot \sin 60}{26.5}\right)$

$B \approx 40.9°$

$\angle C = 180 - 60 - 40.9$

$\angle C = 79.1°$

6.2 Law of Cosines

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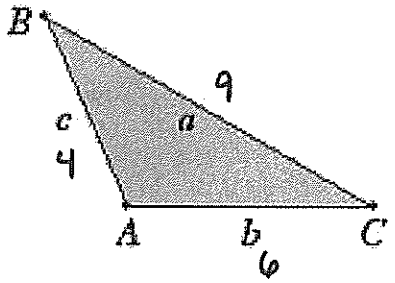
How do you find the missing sides of a triangle when you know the angles of the triangle?

Case 2: *Side-Side-Side (SSS)*

1. Use Law of Cosines to find the angle opposite the longest side.
2. Use the Law of Sines to find either of the two remaining acute angles.
3. Find the third angle by using the fact that the sum of the angles of a triangle is 180°.

Example 2: ***Use the law of cosines to solve the triangle.***



a = 9 $A = 127.2°$

b = 6 $B = 32.1°$

c = 4 $C = 20.7$ $(180 - 127.2 - 32.1)$

$9^2 = 4^2 + 6^2 - 2(4)(6)\cdot\cos A$

$81 = 16 + 36 - 48\cos A$

$29 = -48\cos A$

$\frac{-29}{48} = \cos A$

$\cos^{-1}\left(\frac{-29}{48}\right) = A$

$127.2° \approx A$

$\frac{\sin B}{6} = \frac{\sin 127.2}{9}$

$B = \sin^{-1}\left(\frac{6\cdot\sin 127.2}{9}\right)$

$B = 32.1°$

Day 1 Assignment

6.2 Law of Cosines

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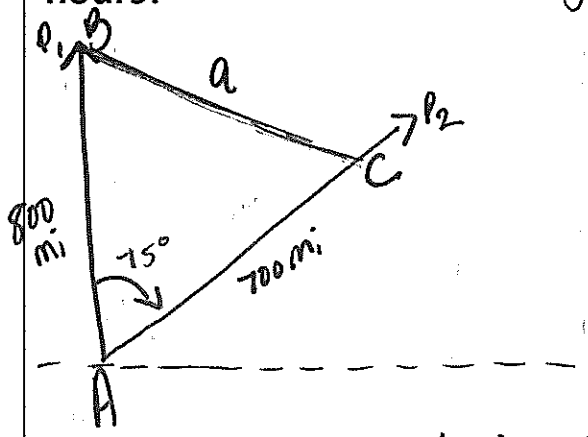
Example 3: Two planes leave an airport at the same time on different runways. One flies directly north at 400 mph. The other airplane flies on a bearing of N75°E at 350 mph. How far apart will the planes be after two hours?

SAS



$a^2 = 800^2 + 700^2 - 2(800)(700)\cos 75$

$a = 916.6\text{mi}$

6.2 Law of Cosines

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Precalculus

Herron's Formula

If a, b, and c are the sides of Δ ABC, and s is the semi-perimeter

S = (a+b+c)/2

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

Example 4: ***Find the area of a triangle with the sides 9, 13, and 17.***

$A = \sqrt{19.5(19.5-9)(19.5-13)(19.5-17)}$ $\quad S = \frac{9+13+17}{2}$

$A = 57.68$ $\quad S \approx 19.5$

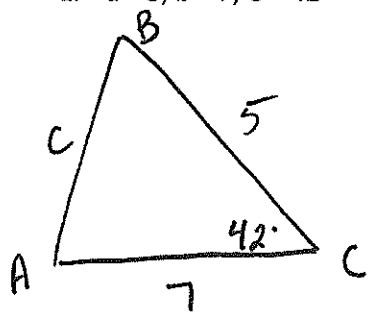
$A \approx 58u^2$

Day 2 Assignment

Law of Cosines Assignment – Day 1

Using the dimensions given sketch the triangle determine which case of *Law of Cosines* is applicable. Then solve the triangle. (*Round dimensions to the nearest tenth.*)

1. a = 5, b = 7, C = 42° SAS



$C^2 = 5^2 + 7^2 - 2(5)(7)\cdot\cos 42°$

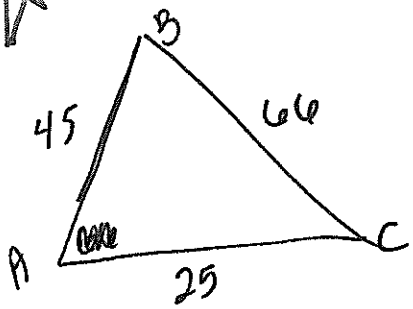
$C = 4.7$

$\frac{\sin B}{7} = \frac{\sin 42}{4.7}$

$B = 85.3°$

$C = 52.7°$

2. a = 66, b = 25, c = 45 SSS



$66^2 = 25^2 + 45^2 - 2(25)(45)\cdot\cos A$

$1706 = -2250\cos A$

$\frac{-1706}{2250} = \cos A$

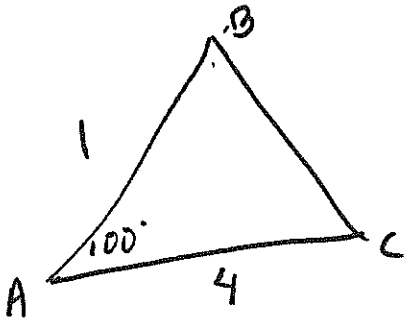
$139° = A$

$\frac{\sin B}{25} = \frac{\sin 139.}{66}$

$B = 14°$

$C = 27°$

3. b = 4, c = 1, A = 100° SAS



$a^2 = 1^2 + 4^2 - 2(1)(4) \cdot \cos 100°$

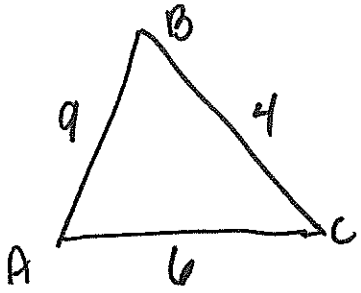
$a = 4.3$

$\frac{\sin B}{4} = \frac{\sin 100}{4.3}$

$B = 66.4°$

$C = 13.6°$

4. a = 4, b = 6, c = 9 SSS



$9^2 = 4^2 + 6^2 - 2(4)(6) \cdot \cos C$

$29 = -48 \cos C$

$\frac{-29}{48} = \cos C$

$127.2° = C$

$\frac{\sin B}{6} = \frac{\sin 127.2}{9}$

$B = 32.1°$

$A = 20.7°$

Law of Cosines Assignment – Day 2

Use Herron's formula to find the area of each triangle. (*Round dimensions to the nearest tenth.)*

1. a = 4 feet, b = 4 feet, c = 2 feet

$S = \frac{4+4+2}{2} = 5$

$A = \sqrt{5(5-4)(5-4)(5-2)}$

$A = 3.9 ft^2$

2. a = 16 meters, b = 10 meters, c = 8 meters

$S = \frac{16+10+8}{2} = 17m$

$A = \sqrt{17(17-16)(17-10)(17-8)}$

$A = 32.7m^2$

3. a = 13 yards, b = 9 yards, c = 5 yards

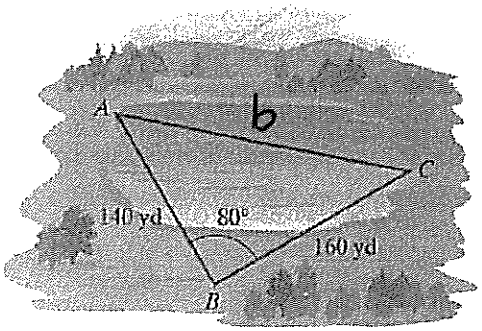
$S = \frac{13+9+5}{2} = \frac{27}{2} = 13.5$

$A = \sqrt{\underset{13.5}{\frac{27}{2}}\left(\underset{13.5}{\frac{27}{2}} - 13\right)(13.5-9)(13.5-5)}$

$A = 16.1 yd^2$

Solve.

4. Find the distance across the Lake from A to C, to the nearest yard , using the measurements shown in the figure. SAS

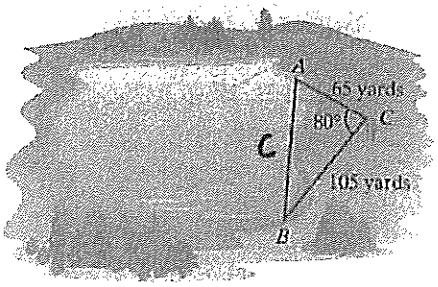


$b^2 = 140^2 + 160^2 - 2(140)(160)\cdot\cos 80°$

$b = 193.44$

$b \approx 194 yd$

5. To find the distance across a protected cove at a lake, a surveyor makes the measurements shown in the figure. Use these measurements to find the distance from *A* to *B* to the nearest yard.

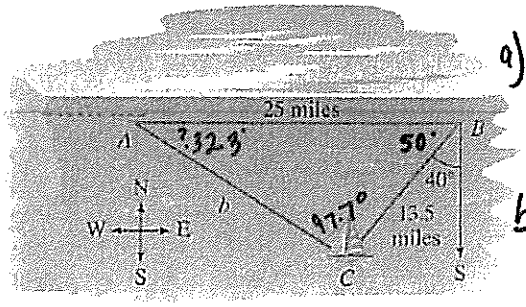


$c^2 = 65^2 + 105^2 - 2(65)(105)\cos 80°$

$c = 113.5$

$c \approx 114 yd$

6. You are on a fishing boat that leaves it pier and heads east. After traveling for 25 miles, there is a report warning rough seas directly south. The captain turns the boat and follows a bearing of S40°W for 13.5 miles.
   a. How far are from the boats pier?
   b. What bearing could the boat have originally taken to arrive at this spot?



a) $b^2 = 25^2 + 13.5^2 - 2(25)(13.5)\cos 50°$

$b = 19.3 mi$

b) S58°E