The Law of Sines

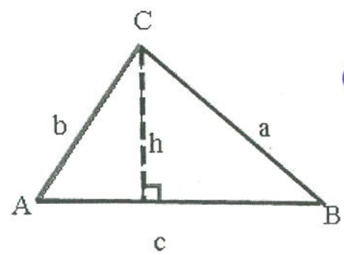
Students will utilize the Law of Sines to find the missing sides and angles of acute and obtuse triangles.

Precalculus

What is the Law of Sines?

Law of Sines

Used to find the missing sides and angles of oblique (non-right) triangles. Right triangle trigonometry won't work.



$\sin B = \frac{h}{a}$

$h = a \cdot \sin B$

$\sin A = \frac{h}{b}$

$h = b \cdot \sin A$

$a \cdot \sin B = b \cdot \sin A$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

In any Δ ABC with angles A, B, and C and opposite sides a,b, and c the following equation is true:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Sines can be used to solve the following cases of oblique triangles:

1. **Angle-Angle-Side: AAS**
2. **Angle-Side-Angle: ASA (side is included)**
3. **Side-Side-Angle: SSA (special case)**

Law of sines can be used for both acute and obtuse triangles.

6.1 The Law of Sines

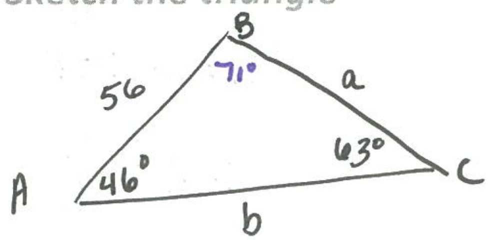
Students will utilize the Law of Sines to find the missing sides and angles of acute and obtuse triangles.

Precalculus

Example 1: AAS

Solve the $\triangle ABC$ if $m\angle A = 46^\circ$, $m\angle C = 63^\circ$, $c = 56$

a. Sketch the triangle



b. Find the measure of angle B.

$\measuredangle B = 180 - 46 - 63$

$\measuredangle B = 71^\circ$

c. Solve for side a and side b. * I MUST use $\frac{c}{\sin C}$ b/c I know side C + angle C.*

Side a

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin 46^\circ} = \frac{56}{\sin 63^\circ}$

$a \cdot \sin 63^\circ = 56 \cdot \sin 46^\circ$

$a = \frac{56 \cdot \sin 46}{\sin 63}$

$a \approx 45.21$

Side b

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{b}{\sin 71^\circ} = \frac{56}{\sin 63^\circ}$

$b \cdot \sin 63 = 56 \cdot \sin 71$

$b = \frac{56 \cdot \sin 71}{\sin 63}$

$b \approx 59.43$

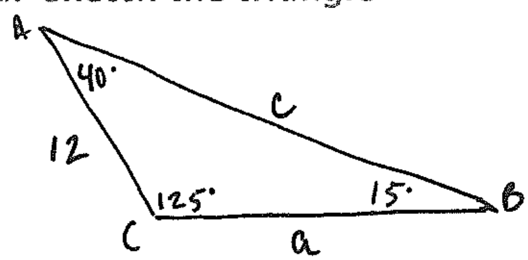
The Law of Sines Students will utilize the Law of Sines to find the missing sides and angles of acute and obtuse triangles.	 ***Precalculus***

Example 2: ASA (side included)

Solve ΔABC if m∠A = 40°, m∠C = 225°, b. = 12

125°

a. Sketch the triangle



b. Find the measure of angle B.

$\measuredangle B = 180 - 125 - 40$

$\measuredangle B = 15°$

c. Solve for side a and side c.

Side a

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 40°} = \frac{12}{\sin 15°}$$

$$a = \frac{12 \cdot \sin 40}{\sin 15}$$

$$a \approx 29.8$$

Side c

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 125} = \frac{12}{\sin 15°}$$

$$C = \frac{12 \cdot \sin 125}{\sin 15}$$

$$c \approx 37.98$$

The Law of Sines

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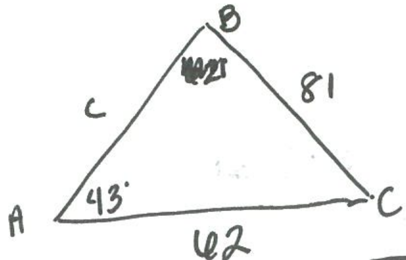
Precalculus

Example 3: SSA ("the ambiguous case")

Could be Acute or Obtuse

Solve $\triangle ABC$ if $m\angle A = 43^\circ$, $a = 81$, $b = 62$

a. Sketch the triangle.



Solutions

1

b. Find the measure of $\boxed{\text{angle B}}$ and determine the number of solutions. b/c I know side b.

$$\cancel{\frac{b}{\sin B} = \frac{a}{\sin A}} \qquad \frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\cancel{\frac{62}{\sin B} = \frac{81}{\sin 43}} \qquad 62 \cdot \frac{\sin B}{62} = \frac{\sin 43^\circ}{81} \cdot 62$$

$$\sin B = \frac{62 \cdot \sin 43}{81}$$

$$B^{th} = \sin^{-1}\left(\frac{62 \cdot \sin 43}{81}\right) \approx \boxed{31.5^\circ}$$

$\measuredangle C = 180 - 43 - 31.5 = \boxed{105.5^\circ}$

2 Δ's?

① Find supp. to 31.5

$180 - 31.5 = 148.5^\circ$

② Add to other angle

$148.5 + 43 = 191.5$

③ If sum is

$> 180 \to 1\ \Delta$

$< 181 \to 2\ \Delta$

c. Solve for side c.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 105.5} = \frac{81}{\sin 43}$$

$$c = \frac{81 \cdot \sin 105.5^\circ}{\sin 43}$$

$\boxed{c = 114.4}$

*

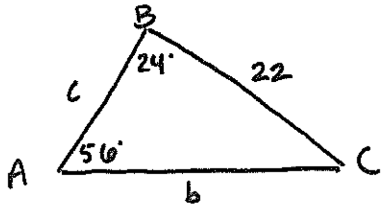
Law of Sines Day 1 Assignment

AAS, ASA, SSA

Using the dimensions given and sketch the triangle and determine which case of *Law of Sines* is applicable. Then solve the triangle. *(Round side lengths to the nearest tenth and angle measures to the nearest degree.)*

1. $A = 56°$ $B = 24°$ $a = 22$

AAS



$\angle C = 180 - 56 - 24 = \boxed{100°}$

$$\frac{b}{\sin 24°} = \frac{22}{\sin 56°}$$

$$22 \cdot \sin 24 = b \cdot \sin 56$$

$$\frac{22 \cdot \sin 24}{\sin 56} = b$$

$$\boxed{10.8 = b}$$

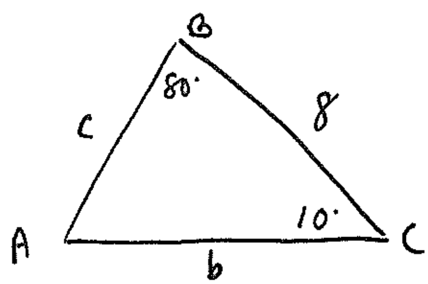
$$\frac{c}{\sin 100°} = \frac{22}{\sin 56}$$

$$\frac{22 \cdot \sin 100}{\sin 56} = c$$

$$\boxed{26.1 = c}$$

2. $B = 80°$ $C = 10°$ $a = 8$

ASA



$\angle A = 180 - 80 - 10 = \boxed{90°}$

$$\frac{b}{\sin 80°} = \frac{8}{\sin 90}$$

$$b \cdot \sin 90 = 8 \cdot \sin 80$$

$$b = \frac{8 \cdot \sin 80}{\sin 90}$$

$$\boxed{b = 7.9}$$

$$\frac{c}{\sin 10°} = \frac{8}{\sin 90°}$$

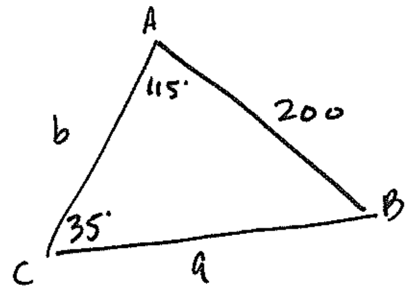
$$c = \frac{8 \cdot \sin 10}{\sin 90}$$

$$\boxed{c = 1.4}$$

3. $A = 115°$ $C = 35°$ $c = 200$ AAS

$\measuredangle B = 180 - 115 - 35$

$\boxed{\measuredangle B = 30°}$



$$\frac{200}{\sin 35°} = \frac{a}{\sin 115°}$$

$$200 \cdot \sin 115 = a \cdot \sin 35$$

$$\frac{200 \cdot \sin 115}{\sin 35} = a$$

$$\boxed{316.0 = a}$$

$$\frac{200}{\sin 35} = \frac{b}{\sin 30}$$

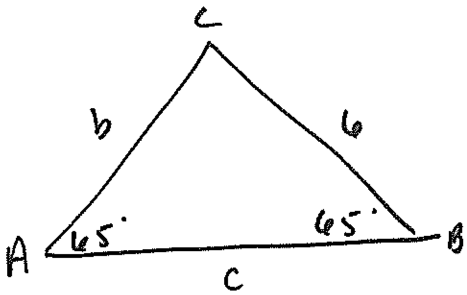
$\vdots$

$$\frac{200 \cdot \sin 30°}{\sin 35°} = b \qquad \boxed{174.3 = b}$$

4. $A = 65°$ $B = 65°$ $a = 6$ AAS

$\measuredangle C = 180 - 65 - 65$

$\boxed{\measuredangle C = 50°}$



$$\frac{6}{\sin 65} = \frac{b}{\sin 65} \qquad \frac{6}{\sin 65} = \frac{c}{\sin 50}$$

$$\boxed{b = 6} \qquad \boxed{c = 5.1}$$

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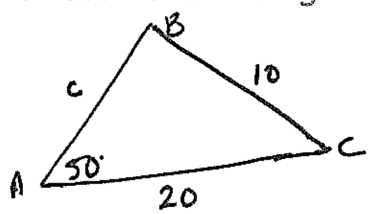
Example 4: SSA ("the ambiguous case")

Solve $\triangle ABC$ if $m\angle A = 50°$, $a = 10$, $b = 20$

a. Sketch the triangle.



b. Find the measure of angle B and determine the number of solutions.

$$\frac{\sin B}{20} = \frac{\sin 50}{10}$$

$$10 \cdot \sin B = 20 \cdot \sin 50$$

$$\sin B = \frac{20 \cdot \sin 50}{10}$$

$$B = \sin^{-1}\left(\frac{20 \cdot \sin 50}{10}\right)$$ undef. $\Rightarrow$ No Δ exists

c. Solve for side c.

NA

The Law of Sines

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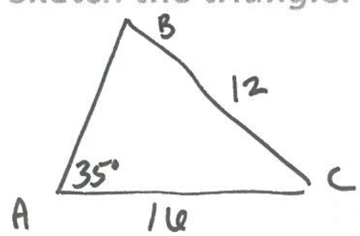
Precalculus

Example 5: SSA ("the ambiguous case")

Solve $\triangle ABC$ if $m\angle A = 35^\circ$, $a = 12$, $b = 16$

a. Sketch the triangle.



2Δ's

$180 - 50 = 130$

$130 + 35 = 165$

$< 180 \Rightarrow 2\Delta$'s

b. Find the measure of angle B and determine the number of solutions.

$$\frac{\sin B}{16} = \frac{\sin 35}{12}$$

$$12 \cdot \sin B = 16 \cdot \sin 35$$

$$B = \sin^{-1}\left(\frac{16 \cdot \sin 35}{12}\right)$$

$$B_1 \approx 50^\circ$$

$$C_1 = 95 \ (180 - 50 - 35)$$

Solution 2

$$B_2 = 130^\circ$$

$$C_2 = 15^\circ \ (180 - 130 - 35)$$

c. Solve for side c.

$$\frac{c_1}{\sin 95} = \frac{12}{\sin 35}$$

$$c_1 = \frac{12 \cdot \sin 95}{\sin 35}$$

$$c_1 = 20.8$$

$$\frac{c_2}{\sin 15} = \frac{12}{\sin 35}$$

$$c_2 = \frac{12 \cdot \sin 15}{\sin 35}$$

$$c_2 = 5.4$$

The Law of Sines

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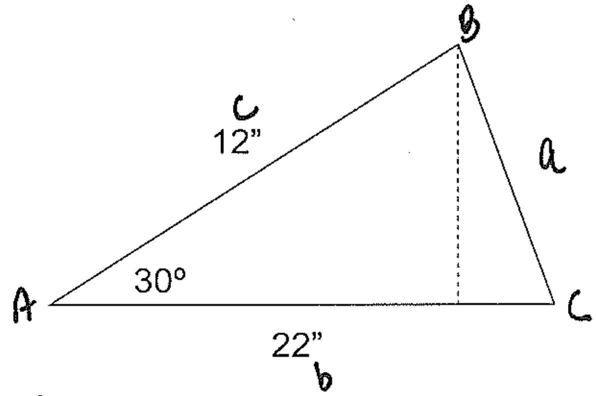
Precalculus

Finding the Area of a Triangle:

$$\text{Area} = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$

Example 6: ***Find the area of the triangle.***



$A = \frac{1}{2}(22)(12)\sin 30°$

$A = 132 \cdot \sin 30$

$A = 66\ u^2$

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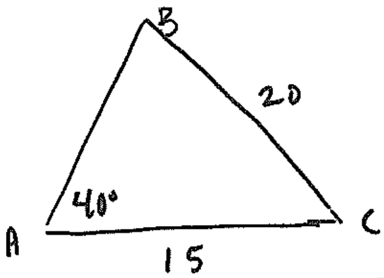
Law of Sines Day 2 Assignment

Using the dimensions given and sketch the triangle and determine which case of *Law of Sines* is applicable. Then solve the triangle. *(Round side lengths to the nearest tenth and angle measures to the nearest degree.)*

1. $a = 20$ $b = 15$ $A = 40°$ SSA



$$\frac{\sin B}{15} = \frac{\sin 40}{20}$$

$$B = \sin^{-1}\left(\frac{15 \cdot \sin 40}{20}\right)$$

$\boxed{\angle B = 28.8°}$

$\boxed{\angle C = 111.2°}$

$$\frac{c}{\sin 111.2} = \frac{20}{\sin 40}$$

$$c = \frac{20 \cdot \sin 111.2}{\sin 40}$$

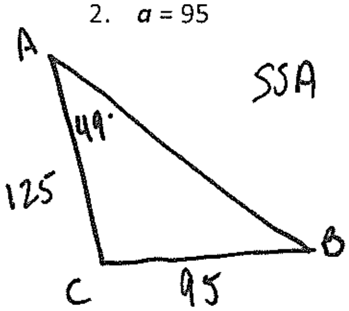
$\boxed{c = 29}$

$180 + 28.8 = 151.2$

$151.2 + 40 = 191.2$

1 △

2. $a = 95$ $b = 125$ $A = 49°$



SSA

$$\frac{\sin B}{125} = \frac{\sin 49}{95}$$

$$B = \sin^{-1}\left(\frac{125 \cdot \sin 49}{95}\right)$$

$B_1 \approx 83.2°$	$C_1 = 180 - 83.2 - 49 = 47.8°$
$B_2 = 96.8°$	$C_2 = 180 - 96.8 - 49 = 34.2°$

$180 - 83.2 = 96.8$

$96.8 + 49 = 145.8$

2 △'s

side c_1

$$\frac{c_1}{\sin 47.8} = \frac{95}{\sin 49}$$

$$c_1 = \frac{95 \cdot \sin 47.8}{\sin 49}$$

$\boxed{c_1 = 93.2}$

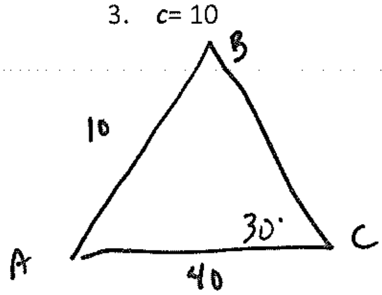
side c_2

$$\frac{c_2}{\sin 34.2} = \frac{95}{\sin 49}$$

$$c_2 = \frac{95 \cdot \sin 34.2}{\sin 49}$$

$\boxed{c_2 = 70.8}$

3. $c = 10$ $b = 40$ $C = 30°$ SSA



$$\frac{\sin B}{40} = \frac{\sin 30}{10}$$

$$B = \sin^{-1}\left(\frac{40 \cdot \sin 30}{10}\right)$$

$B \approx$ undefined

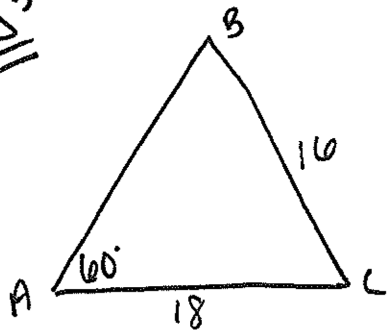
No △ exists

$180 - 77 = 103$

$103 + 60 = 163$

$2\triangle$'s

4. $a = 16$ $b = 18$ $A = 60°$ SSA



$\boxed{\begin{matrix} B_2 \approx 103° \\ C_2 \approx 17° \end{matrix}}$

$$\frac{\sin B}{18} = \frac{\sin 60}{16}$$

$$B = \sin^{-1}\left(\frac{18 \cdot \sin 60}{16}\right)$$

$\boxed{\begin{matrix} B_1 \approx 77° \\ C_1 \approx 43° \end{matrix}}$

Side c_1: $\frac{c_1}{\sin 43} = \frac{16}{\sin 60}$

$$c_1 = \frac{16 \cdot \sin 43}{\sin 60}$$

$\boxed{c_1 \approx 12.6}$

Side c_2: $\frac{c_2}{\sin 17} = \frac{16}{\sin 60}$

$$c_2 = \frac{16 \cdot \sin 17}{\sin 60}$$

$\boxed{c_2 = 5.4}$

Find the area of the triangle having the given measurements. *(Round your answer to the nearest tenth.)*

5. $A = 48°$ $b = 20$ feet $c = 40$ feet

$A = \frac{1}{2} b \cdot c \sin A$

$A = \frac{1}{2}(20)(40) \cdot \sin 48°$

$\boxed{A \approx 297.3 u^2}$

6. $B = 36°$ $a = 3$ yards $c = 6$ yards

$A = \frac{1}{2} a \cdot c \cdot \sin B$

$A = \frac{1}{2}(3)(6) \cdot \sin 36°$

$\boxed{A \approx 5.3 u^2}$

The Law of Sines

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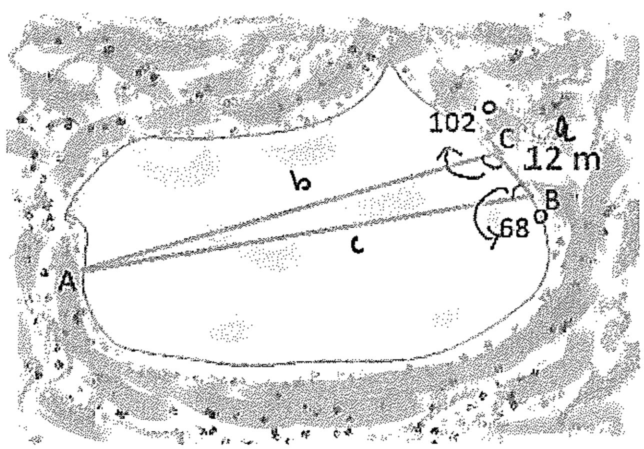
Precalculus

Example 7: Applications

In a children's amusement park, the administration wanted to build a walker's bridge along a Pond in the midst of the park from point A to B. For this purpose an Engineer identified the approximate straight edge of the Pond near B and marked a point C at a distance 12 m from B. He was able to measure the angles ACB and ABC as 102° and 68° to estimate the length of the bride to be built. Find the approximate length of the bridge rounded to a meter.

$\measuredangle A = 180 - 102 - 68$

$\underline{\measuredangle A = 10°}$



$$\frac{c}{\sin 102} = \frac{12}{\sin 10}$$

$$C = \frac{12 \cdot \sin 102}{\sin 10}$$

$$C = 67.59 \approx \boxed{68m}$$

The Law of Sines

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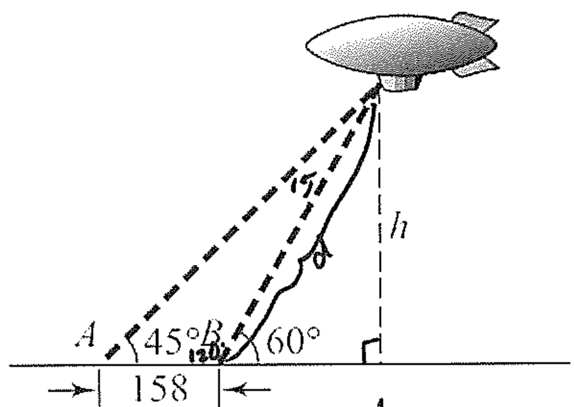
Precalculus

Example 8: Applications

Height of Blimp In order to find the height of the Goodyear blimp, observers at A and B, 158 yards apart, measure the following angles: $\alpha = 45.0°$ and $\beta = 60.0°$. (See the diagram.) How high is the blimp?



$$\frac{\sin 45}{d} = \frac{\sin 15}{158}$$

$$\frac{d}{\sin 45} = \frac{158}{\sin 15}$$

$$d = \frac{158 \cdot \sin 45}{\sin 15}$$

$$d = 431.6$$

$$\cancel{\frac{h}{\sin 60}}$$

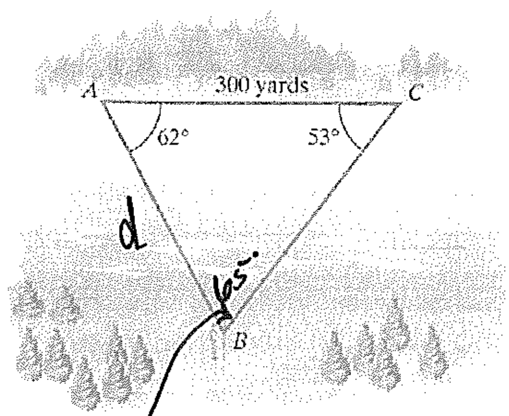
$$\sin 60 = \frac{h}{431.6}$$

$$431.6 \cdot \sin 60 = h$$

$$373.77 \text{ yds} = h$$

$$\boxed{\approx 374 \text{ yds}}$$

A surveyor needs to determine the distance between two points that lie on opposite banks of a river. The figure shows that 300 yards are measured along one bank. The angles from each end of this line segment to a point on the opposite bank are 62° and 53°. Find the distance between ***A*** and ***B*** to the nearest tenth of a yard.



$180 - 62 - 53 = 65°$

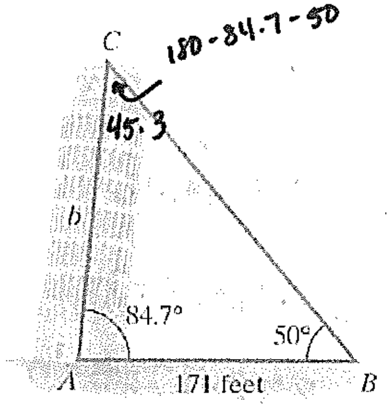
$$\frac{d}{\sin 53} = \frac{300}{\sin 65}$$

$$d = \frac{300 \cdot \sin 53}{\sin 65}$$

$$\boxed{d = 264.4 \text{ yds}}$$

The Leaning Tower of Pisa in Italy leans at an angle of 84.7°. The figure shows that 171 feet from the base of the tower, the angle of elevation to the top is 50°. Find the distance, to the nearest tenth of a foot, from the base to the top of the tower.

$180 - 84.7 - 50$

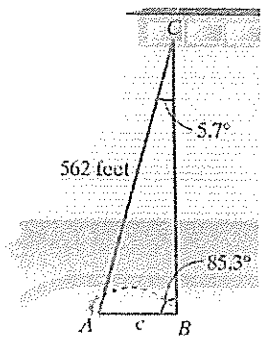


$$\frac{b}{\sin 50} = \frac{171}{\sin 45.3}$$

$$b = \frac{171 \cdot \sin 50}{\sin 45.3}$$

$$\boxed{b = 184.4 \text{ ft.}}$$

The figure shows a shot-put ring. The shot is tossed from ***A*** and lands at ***B***. Using modern electronic, the distance of the toss can be measured without the use of measuring tapes. When the shot lands at ***B***, an electronic transmitter placed at ***B*** sends a signal to a device in the official's booth above the track. The device determines the angles at ***B*** and **C**. At a track meet, the distance from the official's booth to the shot-put ring is 562 feet. If ***B*** = 85.3° and ***C*** = 5.7°, determine the length of the toss to the nearest tenth of a foot.

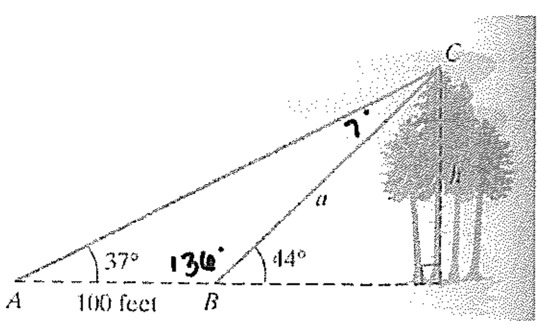


$$\frac{c}{\sin 5.7} = \frac{\cancel{8}\,562}{\sin 85.3}$$

$$C = \frac{562 \cdot \sin 5.7}{\sin 85.3}$$

$$\boxed{C = 56 \text{ ft.}}$$

Redwood trees in California's Redwood National Park are hundreds of feet tall. The height of one of these trees is represented by ***h*** in the figure below. A.) Use the measurements shown to find a, to the nearest tenth of a foot, in oblique triangle ***ABC***. B.) Use the right triangle shown to find the height, to the nearest tenth of a foot, of a typical redwood tree in the park.



A) $\frac{a}{\sin 37} = \frac{100}{\sin 7}$

$$a = \frac{100 \cdot \sin 37}{\sin 7}$$

$$a \approx 493.8 \text{ ft.}$$

B) $\sin 44 = \frac{h}{493.8}$

$$493.8 \cdot \sin 44 = h$$

$$343.04 \approx h$$

$$\boxed{343 \text{ ft.}}$$