

Name: Key

Date: \_\_\_\_\_ Per. : \_\_\_\_\_

Honors Precalculus  
Solving Trigonometric Equations  
Review

Level 1

1. Find the exact value of each expression.

a.  $\sin 315^\circ$

$$\sin(270 + 45) = \sin 270 \cos 45 + \cos 270 \sin 45$$
$$= 1 \cdot \frac{\sqrt{2}}{2} + 0 \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{-\sqrt{2}}{2}}$$

b.  $\tan 15^\circ$

~~$\tan(60-45)$~~

$$\tan(45-30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \cdot \frac{\sqrt{3}}{3}} = \boxed{\frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}}$$

c.  $\cos \frac{5\pi}{12}$

$\frac{1\pi}{12} - \frac{4\pi}{12}$   
 $\frac{3\pi}{4} - \frac{\pi}{3}$

$$\cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) = \cos \frac{3\pi}{4} \cdot \cos \frac{\pi}{3} + \sin \frac{3\pi}{4} \cdot \sin \frac{\pi}{3}$$
$$= \frac{-\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2} + \sqrt{6}}{4}}$$

2. Write the expression as the tangent of an angle, then find the exact value.

a.  $\frac{\tan 155 + \tan 85}{1 - \tan 155 \tan 85} = \tan(155 + 85) = \tan 240 = \sqrt{3}$

b.  $\sin \frac{15\pi}{36} \cos \frac{\pi}{6} - \cos \frac{15\pi}{36} \sin \frac{\pi}{6} = \sin\left(\frac{15\pi}{36} - \frac{\pi}{6}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\frac{15\pi}{36} - \frac{6\pi}{36} = \frac{9\pi}{36}$

c.  $\cos \frac{2\pi}{3} \cos \frac{\pi}{6} + \sin \frac{2\pi}{3} \sin \frac{\pi}{6} = \cos\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = \cos \frac{\pi}{2} = 0$

$\frac{4\pi}{6} - \frac{\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$

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Level 2

3. Solve the trigonometric equation for all values of  $x$  over the interval  $[0, 2\pi]$

a.  $3 \tan x + 6 = 0$

$$3 \tan x = -6$$

$$\tan x = -2$$

No solution

b.  $5 \cot \theta = \sqrt{3} + 4 \cot \theta$

$$\cot \theta = \sqrt{3}$$

$$\cot \theta = \frac{\sqrt{3}}{1}$$

$$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$$

c.  $12 \sin^2 x - 3 = 6 \sin^2 x$

$$6 \sin^2 x = 3$$

$$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$$

$$\sin x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

d.  $4 \cos^2 x - 3 = 0$

$$4 \cos^2 x = 3$$

$$\sqrt{\cos^2 x} = \sqrt{\frac{3}{4}}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

e.  $4 \tan^2 x - 1 = \tan^2 x$

$$3 \tan^2 x = 1$$

$$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$$

$$\tan x = \pm \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

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Level 3

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4. Solve the trigonometric equation for all values of x.

a.  $2\tan^2\theta \sin\theta = \frac{2}{\cot^2\theta}$

$$\begin{aligned} 2\tan^2\theta \sin\theta &= 2\tan^2\theta \\ 2\tan^2\theta(\sin\theta - 1) &= 0 \\ 2\tan^2\theta = 0 &\quad \sin\theta - 1 = 0 \\ \tan^2\theta = 0 &\quad \sin\theta = 1 \\ \tan\theta = 0 & \\ \theta = 0, \frac{\pi}{2}, \pi \} + 2n\pi \end{aligned}$$

b.  $\sec^2\theta - 1 = 2\tan\theta - 1$

$$\begin{aligned} 1 + \tan^2\theta - 1 - 2\tan\theta + 1 &= 0 \\ \tan^2\theta - 2\tan\theta + 1 &= 0 \\ (\tan\theta - 1)^2 &= 0 \\ \tan\theta = 1 & \\ \theta = \frac{\pi}{4}, \frac{5\pi}{4} \} + 2n\pi \end{aligned}$$

c.  $6\cos 2\theta - 1 = 4\cos 2\theta$

$$\begin{aligned} 2\cos 2\theta &= 1 \\ \cos 2\theta &= \frac{1}{2} \\ 2\theta &= \frac{\pi}{3}, \frac{5\pi}{3} \\ \theta &= \frac{\pi}{6}, \frac{5\pi}{6} + 2n\pi \end{aligned}$$

d.  $\csc^2 x - \cot x \csc^2 x = 0$

$$\begin{aligned} \csc^2 x(1 - \cot x) &= 0 \\ \csc^2 x = 0 &\quad \cot x = 1 \\ \csc x = 0 &\quad \tan x = 1 \\ \sin x = \frac{1}{0} & \\ \text{und} & \\ x = \frac{\pi}{4}, \frac{5\pi}{4} \} + 2n\pi \end{aligned}$$

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e.  $\cot^2 x - \sec x \cot^2 x = 0$

$$\cot^2 x (1 - \sec x) = 0$$

$$\cot^2 x = 0 \quad 1 - \sec x = 0$$

$$\cot x = 0 \quad \sec x = 1$$

$$\tan x \text{ und } \cos x = 1$$

$$x = 0 + 2n\pi$$

f.  $1 - \cos^2 \theta = \sin \theta + 2$

$$\sin^2 \theta - \sin \theta - 2 = 0$$

$$(\sin \theta - 2)(\sin \theta + 1)$$

$$\sin \theta = 2 \quad \sin \theta = -1$$

↓  
NO sol'n

$$\theta = \frac{3\pi}{2} + 2n\pi$$

g.  $2\sin^4 x - 3\sin^2 x = -1$

$$(2\sin^2 x - 1)(\sin^2 x - 1)$$

$$\sin^2 x = \frac{1}{2} \quad \sin^2 x = 1$$

$$\sin x = \pm \frac{\sqrt{2}}{2}, \pm 1$$

$$x = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2} \right\} + 2n\pi$$