Graphs of Rational Functions • *Students will be able to analyze rational functions and sketch their graphs.*	
	RPC/HPC

What is an rational function?

If $N(x)$ and $D(x)$ are functions with $D(x) \neq 0$,

$r(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1}x^{n-1}}{b_m x^m + b_{m-1}x^{m-1}}$ **is a Rational Function**

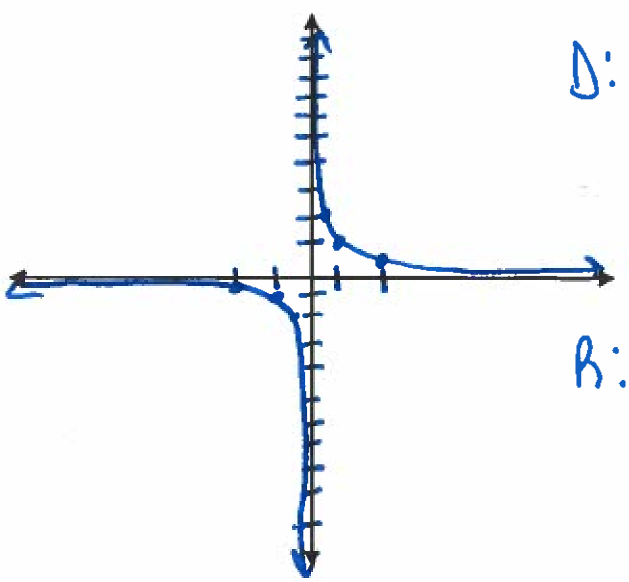
Sketch the graph of $f(x) = \frac{1}{x}$.

State the Domain and Range.

x	2	1	0.5	0.1	0.01	0.001	$\to 0$
$f(x)$	$\frac{1}{2}$	1	2	10	100	1000	$+\infty$

x	-2	-1	-0.5	-0.1	-0.01	-0.001	$\to 0$
$f(x)$	$-\frac{1}{2}$	-1	-2	-10	-100	-1000	$-\infty$

What is the Domain of f(x)?



D: $(-\infty, 0) \cup (0, \infty)$

What is the Range of f(x)?

R: $(-\infty, 0) \cup (0, \infty)$

Section 2.6: Graphs of Rational Functions • *Students will be able to analyze rational functions and sketch their graphs.*	 Honors PreCalculus

Can we find asymptotes of rational functions without graphing?

$r(x) = \frac{a_n x^n}{b_m x^m}$

Two basic types of asymptotes:

- Vertical
  - Indicates a restriction on the $\boxed{\text{domain}}$ of a function
  - $\lim_{x \to a} f(x) = \infty$ or $\lim_{x \to a} f(x) = -\infty$
- End Behavior: Can be horizontal, slant or a non-linear function
  - Indicates a restriction on the $\boxed{\text{range}}$ of a function
  - $\lim_{x \to \infty} f(x) = b$ or $\lim_{x \to -\infty} f(x) = b$

Finding asymptotes:

Vertical: Occur at zeros of the denominator

End Behavior: Compare the degrees of the numerator (n) and denominator (d)

- If n < m, the asymptote is horizontal; $y = 0$
- If n = m, the asymptote is horizontal; $y = \frac{a_n}{b_m}$
- If n > m, end behavior asymptote is found by actually dividing the polynomials ex 2

x-intercepts (a.k.a. "solutions") occur at the zeros of the numerator

X-intercepts = roots = zeros = Solutions
(all the same)

Example 1: Find any asymptotes and the Domain for $f(x)=\dfrac{4}{(x-2)^3}$

Vertical Asymptote(s):

$\sqrt[3]{(x-2)^3}=\sqrt[3]{0}$

$x-2=0$

$x=2$

Domain:

$(-\infty,2)\cup(2,\infty)$

$\mathbb{R},\ x\neq 2$

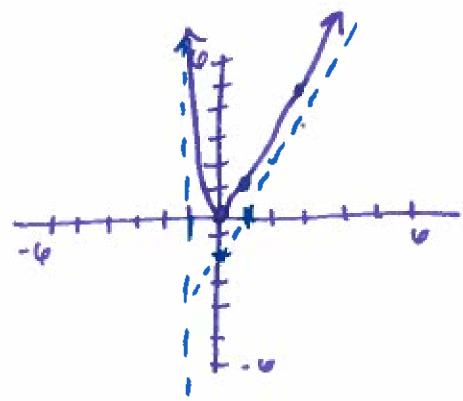
End Behavior Asymptote(s): $y=0$

~~Num: degree 0~~

$n:0 \quad m:3 \quad n<m$

H.A. $y:0$

Example 2: Find any asymptotes and the Domain for $f(x)=\dfrac{2x^2}{x+1}$



Vertical Asymptote(s):

$x+1=0$

$x=-1$

Domain:

$(-\infty,-1)\cup(-1,\infty)$

End Behavior Asymptote(s):

$n=2 \quad m=1$

$n>m$

EB is slant asym.

$y=2x-2$

$$
\begin{array}{r|l}
 & 2x-2+\frac{2}{x+1} \\
x+1 & 2x^2 \\
 & -2x^2+2x \\
\hline
 & -2x+0 \\
 & -(-2x-2) \\
\hline
 & +2
\end{array}
$$

Show graph

Example 3: Find any asymptotes and the Domain for $f(x)=\dfrac{2x^2}{(x^2-4)}$

Vertical Asymptote(s):

$x^2-4=0$

$x^2=4$

$x=\pm 2$

Domain:

$(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

$\mathbb{R},\ x\neq -2,\ x=2$

End Behavior Asymptote(s):

$n=2 \quad m=2 \quad n=m$

H.A. $y=\dfrac{a}{b}=\dfrac{2}{1}=2$

Example 4: Find any asymptotes and the Domain for $f(x)=\dfrac{x^3-3x^2+3x+1}{x-1}$

Vertical Asymptote(s):

$x-1=0$

$x=1$

Domain:

$(-\infty,1)\cup(1,\infty)$

End Behavior Asymptote(s):

$n=3 \quad m=1 \quad n>m$

Parabolic asym. at $y=x^2-2x+1$

Long Div.

$$
\begin{array}{r|l}
 & x^2-2x+1+\frac{2}{x-1} \\ \hline
x-1 & x^3-3x^2+3x+1 \\
 & -x^3-x^2 \\ \hline
 & -2x^2+3x \\
 & -(-2x^2+2x) \\ \hline
 & x+1 \\
 & -(x-1) \\ \hline
 & 2
\end{array}
$$

Synth. Div.

$$
\begin{array}{c|cccc}
1 & 1 & -3 & 3 & 1 \\
+ & \downarrow & 1 & -2 & 1 \\ \hline
 & 1 & -2 & 1 & 2
\end{array}
$$

$\boxed{x^2-2x+1+\frac{2}{x-1}}$