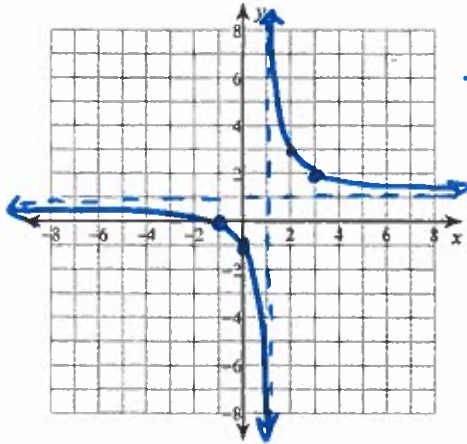


Graphing Rational Functions

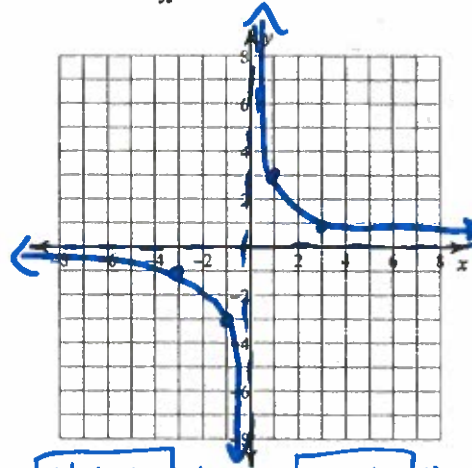
Graph each function. Identify all key features of the function.

1)  $f(x) = \frac{2}{x-1} + 1 = \frac{2}{x-1} + \frac{x-1}{x-1} = \frac{x+1}{x-1}$



| x  | f(x)       |
|----|------------|
| -2 | -1/3 = 0.3 |
| 2  | 3/1 = 3    |
| 3  | 4/2 = 2    |
| 4  | 5/3        |

2)  $f(x) = \frac{3}{x}$



| x  | f(x) |
|----|------|
| -3 | -1   |
| -1 | -3   |
| 1  | 3    |
| 3  | 1    |

**X-int**  $x = -1$   
 $x+1=0$

**y-int**  $y = -1$   
 $y = \frac{0+1}{0-1} = -1$

V.A.  $x-1=0$   
 $x=1$

E.B.  $n=1$   $m=1$   
 $y=1$

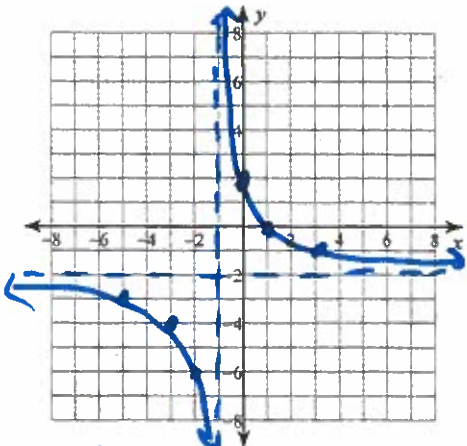
**X-int** None  
 $3=0$

**y-int** None  
 $y = \frac{3}{0}$

V.A.  $x=0$

E.B.  $n=0$   $m=1$   
 $y=0$

3)  $f(x) = \frac{4}{x+1} - 2 = \frac{4}{x+1} - \frac{2(x+1)}{x+1} = \frac{-2x+2}{x+1}$



| x  | f(x) |
|----|------|
| -5 | -3   |
| -3 | -4   |
| -2 | -6   |
| 0  | 2    |
| 1  | 0    |
| 3  | -1   |

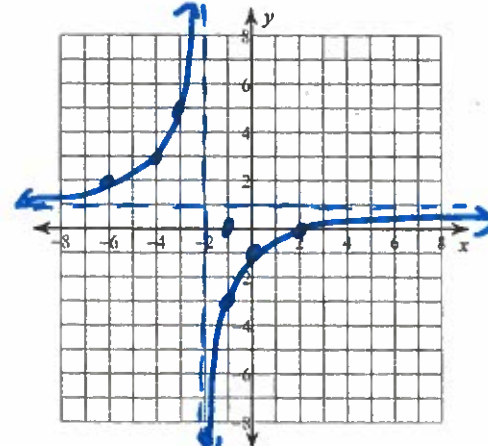
**X-int**  $x = 1$   
 $-2x+2=0$

**y-int**  $y = 2$   
 $y = \frac{-2(0)+2}{0+1} = \frac{2}{1}$

V.A.  $x+1=0$   
 $x=-1$

E.B.  $n=1$   $m=1$   
 $y = \frac{-2}{1}$   
 $n=m$

4)  $f(x) = -\frac{4}{x+2} + 1 = \frac{-4}{x+2} + \frac{x+2}{x+2} = \frac{x-2}{x+2}$



| x  | f(x) |
|----|------|
| -6 | 2    |
| -4 | 3    |
| -3 | 5    |
| -1 | -3   |
| 0  | -1   |

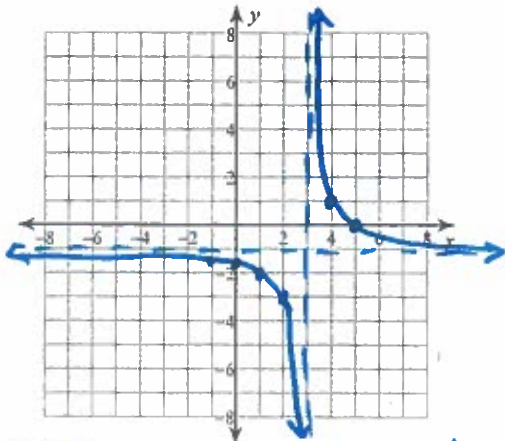
**X-int**  $x = 2$   
 $x-2=0$

**y-int**  $y = -1$   
 $y = \frac{0-2}{0+2} = \frac{-2}{2}$

V.A.  $x+2=0$   
 $x=-2$

E.B.  $n=1$   $m=1$   
 $y=1$

$$5) f(x) = \frac{2}{x-3} - 1 = \frac{2}{x-3} - \frac{x-3}{x-3} = \frac{-x+5}{x-3}$$



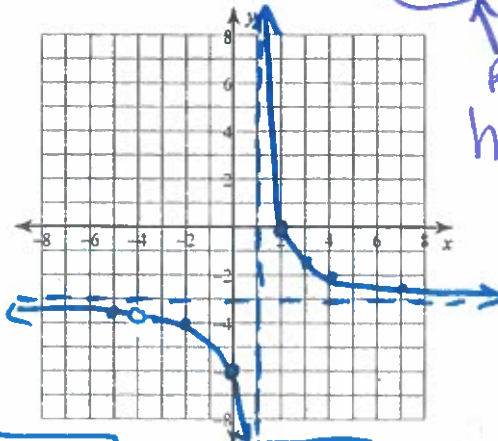
**X-int**  $x=5$   
 $-x+5=0$   
 $-x=-5$   
 $x=5$

**y-int**  $y = \frac{-5}{3} = -1.7$   
 $y = \frac{0+5}{0-3} = \frac{-5}{3}$

V.A.  $x-3=0$   
 $x=3$

E.B.  $n=1$   $m=1$   $n=m$   
 $y = \frac{-1}{1} = -1$

$$6) f(x) = \frac{-3x^2 - 6x + 24}{x^2 + 3x - 4} = \frac{-3(x^2 + 2x - 8)}{(x+4)(x-1)} = \frac{-3(x+4)(x-2)}{(x+4)(x-1)}$$



Reduces out,  
hole @  $x=-4$ .

**X-int**  $x=2$   
 $-3(x-2)=0$   
 $x-2=0$   
 $x=2$

**y-int**  $y=-6$   
 $y = \frac{-3(0-2)}{(0-1)} = \frac{6}{-1}$

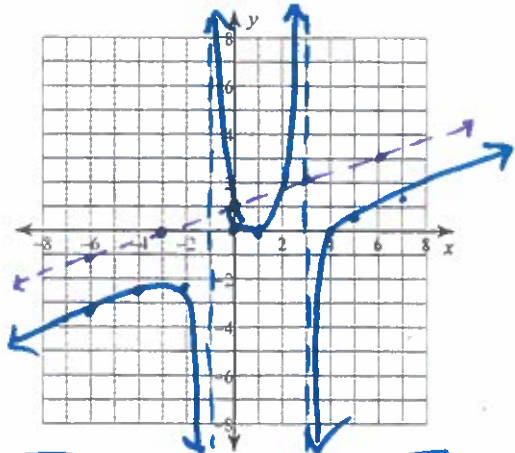
V.A.  $x-1=0$   
 $x=1$

E.B.  $n=1$   $m=1$   $n=m$   
 $y = \frac{-3}{1} = -3$

$$7) f(x) = \frac{x^3 - 5x^2 + 4x}{3x^2 - 6x - 9} = \frac{x(x^2 - 5x + 4)}{3(x^2 - 2x - 3)} = \frac{x(x-4)(x-1)}{3(x-3)(x+1)}$$

$$f(x) = -\frac{2x}{x^2 - 2x} = \frac{2x}{x(x-2)}$$

Reduces out,  
hole @  $x=0$



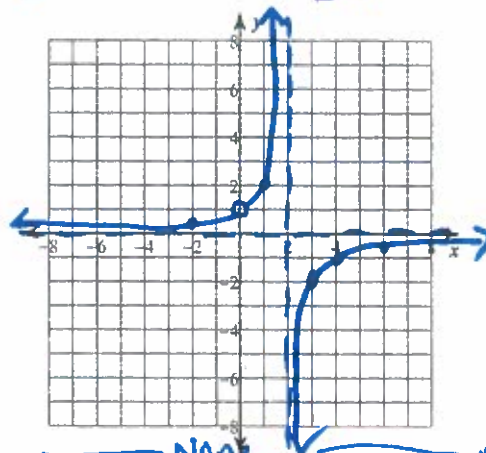
**X-int**  $x=0, x=4, x=1$   
 $x=0$   $x-4=0$   $x-1=0$

**y-int**  $y=0$   
 $y = \frac{0}{3}$

V.A.  $x-3=0$   $x+1=0$   
 $x=3$   $x=-1$

**Long Division!**  
 EB  $n=3$   $n=2$   $n>m$   

$$\begin{array}{r} 43x+1 \\ 3x^2-6x-9 \overline{) x^3-5x^2+4x+0} \\ -(x^3-2x^2+3x) \\ \hline 3x^2+7x+0 \\ -(3x^2-6x-9) \\ \hline 13x-9 \end{array}$$
 remainder  $-2$   
 $y = \frac{1}{3}x + 1$



**X-int** None  
 $x=0$   $x-2=0$

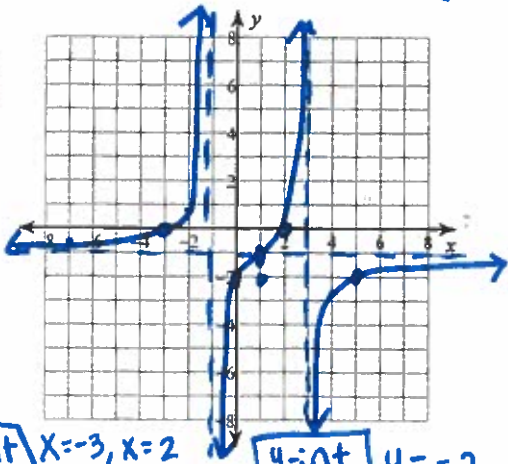
**y-int**  $y=1$   
 $y = \frac{-2}{0-2} = \frac{-2}{-2} = 1$

\*Remember...  $x=0$  is a hole

V.A.  $x-2=0$   
 $x=2$

E.B.  $n=1$   $m=2$   $n<m$   
 $y=0$

$$9) f(x) = \frac{-1(x^2+x-6)}{x^2-2x-3} = \frac{-1(x+3)(x-2)}{(x-3)(x+1)}$$



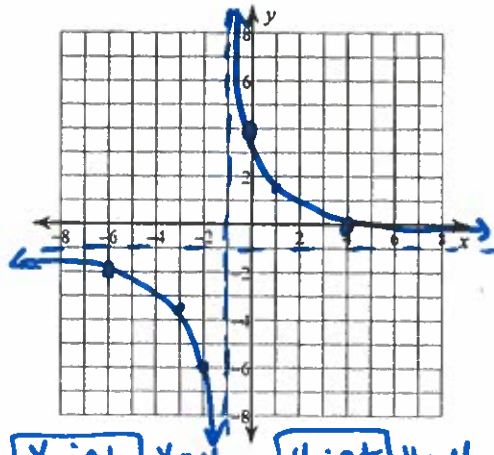
**X-int**  $x = -3, x = 2$   
 $x+3=0 \quad x-2=0$

**y-int**  $y = -2$   
 $\frac{0^2-0+6}{0^2-0-3} = \frac{6}{-3}$

V.A.  $x-3=0 \quad x+1=0$   
 $x=3 \quad x=-1$

E.B.  $n=2 \quad m=2$   
 $n=m$   
 $y = -1$

$$10) f(x) = \frac{x-4}{-x-1}$$



**X-int**  $x = 4$

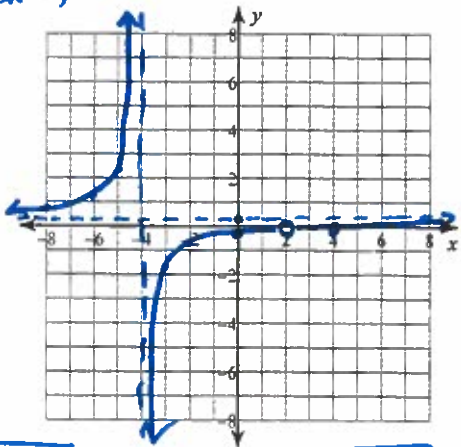
**y-int**  $y = 4$   
 $\frac{0-4}{0-1} = \frac{-4}{-1}$

V.A.  $-x-1=0$   
 $-x=1$   
 $x=-1$

E.B.  $n=1 \quad m=1 \quad n=m$   
 $y = -1$

$$11) f(x) = \frac{x^2-6x+8}{4x^2+8x-32} = \frac{(x-4)(x-2)}{4(x+4)(x-2)}$$

*Hole @ x=2*



**X-int**  $x = 4$   
 $x-4=0$

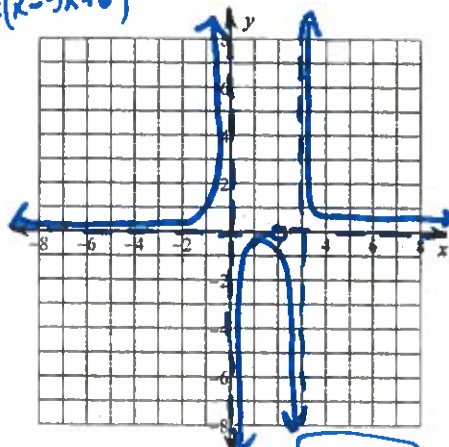
**y-int**  $y = \frac{-1}{4}$   
 $y = \frac{0^2-0+8}{0^2+0-32} = \frac{8}{-32}$

V.A.  $x+4=0$   
 $x=-4$

E.B.  $n=2 \quad m=2$   
 $n=m$   
 $y = \frac{1}{4}$

$$12) f(x) = \frac{x-2}{2x^3-10x^2+12x} = \frac{x-2}{2x(x-3)(x-2)}$$

*Hole @ x=2*



**X-int**  $x = 2$   
 $x-2=0$   
 $x=2$

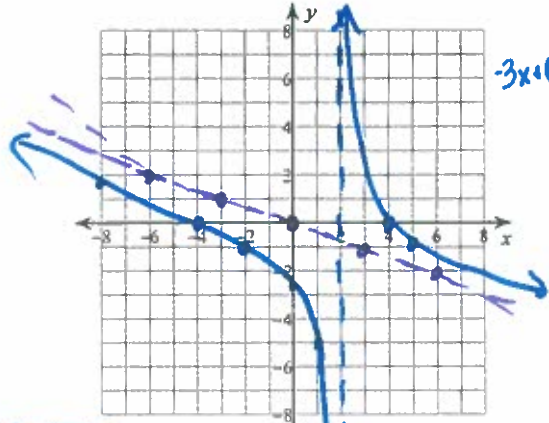
**y-int** None  
 $y = \frac{0-2}{2(0)-10(0)+12(0)} = \frac{-2}{0}$

V.A.  $x=0, x=3$   
 $2x=0 \quad x-3=0$

E.B.  $n=1 \quad m=3 \quad n < m$   
 $y = 0$

4 skip

$$13) f(x) = \frac{x^2 - 16}{-3x + 6} = \frac{(x+4)(x-4)}{-3(x-2)}$$



$$\begin{array}{r} -\frac{1}{3}x - \frac{2}{3} \\ 3x + 6 \overline{) X^2 + 0X - 16} \\ \underline{-(X^2 - 2X)} \phantom{-16} \\ 2X - 16 \\ \underline{-(2X - 4)} \\ -12 \end{array}$$

X-int  $x=4, x=-4$   
 $x^2 - 16 = 0$   
 $x^2 = 16$   
 $x = \pm 4$

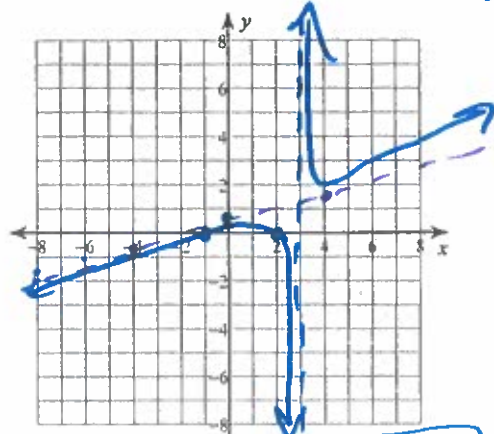
Y-int  $y = -\frac{8}{3} = 2.67$   
 $y = \frac{0 - 16}{0 + 6} = \frac{-16}{6}$

V.A.  $-3x + 6 = 0$   
 $-3x = -6$   
 $x = 2$

E.B.  $n=2, m=1, n > m$   
 $y = \frac{1}{3}x - \frac{2}{3}$

4 skip

$$14) f(x) = \frac{x^2 - x - 2}{4x - 12} = \frac{(x-2)(x+1)}{4(x-3)}$$



X-int  $x=2, x=-1$   
 $x-2=0, x+1=0$

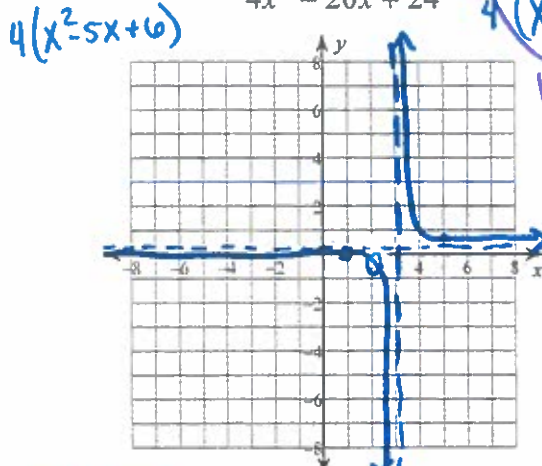
Y-int  $y = \frac{1}{6}$   
 $y = \frac{0^2 - 0 - 2}{0 - 12} = \frac{-2}{-12}$

V.A.  $4x - 12 = 0$   
 $4x = 12$   
 $x = 3$

E.B.  $n=2, m=1, n > m$   
 $\frac{1}{4}x + \frac{1}{2}$   
 $4x - 12 \overline{) X^2 - X - 2}$   
 $\underline{-(X^2 - 3X)}$   
 $2x - 2$   
 $\underline{-(2x + 6)}$   
 $-8$

$y = \frac{1}{4}x + \frac{1}{2}$

15)  $f(x) = \frac{x^2 - 3x + 2}{4x^2 - 20x + 24} = \frac{(x-2)(x-1)}{4(x-2)(x-3)}$



Hole @  $x=2$

X-int  ~~$x=2, x=1$~~   
 $x-2=0, x-1=0$

Y-int  $y = 1/12$   
 ~~$y = \frac{2}{24} = \frac{1}{12}$~~

V.A.  $x-3=0$   
 $x=3$

E.B.  $n=2, m=2, n=m$   
 $y = \frac{1}{4}$