

HPC/RPC Graphing Rational Functions NOTES

Name KEY

Date _____ Per _____

Day 1 - ex 1 + 2

Day 2 - ex 3, 4 + Prob. 6 on W.S.

The steps given below are going to help with graphing rational functions.

- 1) Identify x- and y-intercepts (if there are any).
 numerator = 0 → x = 0
- 2) Find the vertical asymptote(s) by setting the denominator equal to 0 and solving.
- 3) Identify any holes in the graph (factors in the denominator that reduce out).
- 4) Find the end behavior asymptote (if it exists).
- 5) Graph asymptotes using dashed lines.
- 6) Pick some points that you can plot to get the general shape of the graph.

Example 1: Graph the function and identify all key features of the graph.

$$f(x) = \frac{4}{x-2}$$

1) x-intercept(s)? None

(x-value when y=0)

$$0 = \frac{4}{x-2}$$

y-intercept? -2

(y-value when x=0)

$$y = \frac{4}{0-2} = \frac{4}{-2}$$

2) Vertical Asymptote(s) x=2

$$x-2=0$$

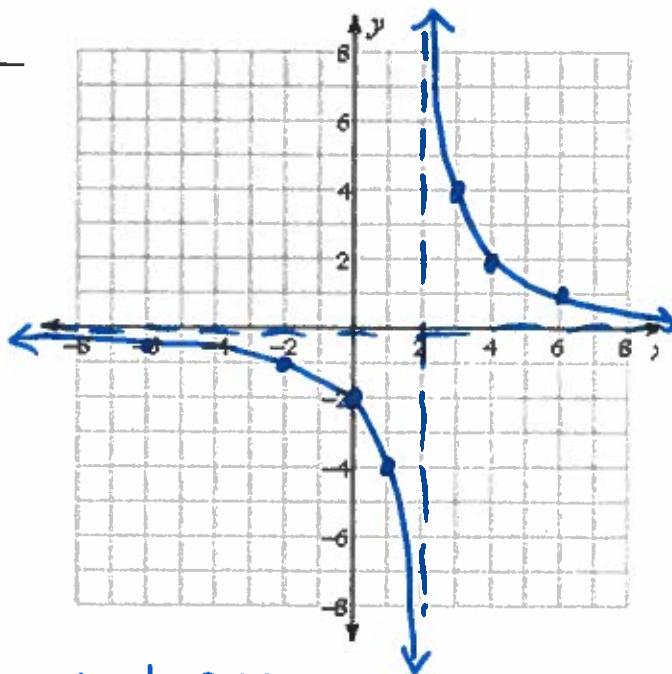
$$x=2$$

3) Holes? NA

4) End Behavior Asymptote y=0

$$n=0 \quad m=1$$

$$n < m$$



x	f(x)
-2	-1
0	-2
-6	-1/2
1	-4
4	2
6	1
3	4

$$-\frac{2}{x-1} + 1 = \frac{-2}{x-1} + \frac{x-1}{x-1} = \frac{x-3}{x-1}$$

Example 2: Graph the function and identify all key features of the graph.

$$f(x) = -\frac{2}{x-1} + 1$$

This moves your E.O. up 1

1) x-intercept(s)? $X=3$ $x-3=0$ $x=3$ y-intercept? 3

$$0 = \frac{-2}{x-1} + 1 \rightarrow -x+1 = -2$$

$$y = \frac{-2}{0-1} + 1 = \frac{-2}{-1} + 1 = 2+1$$

$$-1 = \frac{-2}{x-1} \rightarrow -x+1 = -2$$

$$-x = -3$$

$$x = 3$$

$$-1(x-1) = -2$$

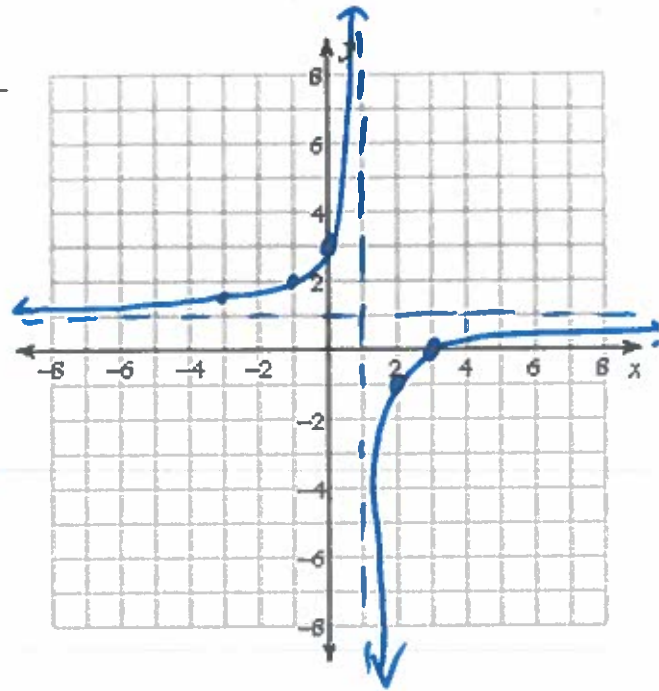
2) Vertical Asymptote(s) $X=1$

$$x-1=0$$

$$x=1$$

3) Holes? None

x	f(x)
-3	1.5
-1	2
0	3
1	undef.
3	0
2	-1



4) End Behavior Asymptote $y=0$ $y=1$ $n=0$ $m=1$ $n < m$
 $n=1$ $m=1$ $n=m$

Example 3: Graph the function and identify all key features of the graph.

$$f(x) = \frac{3}{x^2+x-2} = \frac{3}{(x+2)(x-1)}$$

1) x-intercept(s)? None

y-intercept? $-\frac{3}{2} = -1.5$

$$0 = \frac{3}{x^2+x-2} \quad 3=0$$

$$y = \frac{3}{0+0-2} = \frac{-3}{2}$$

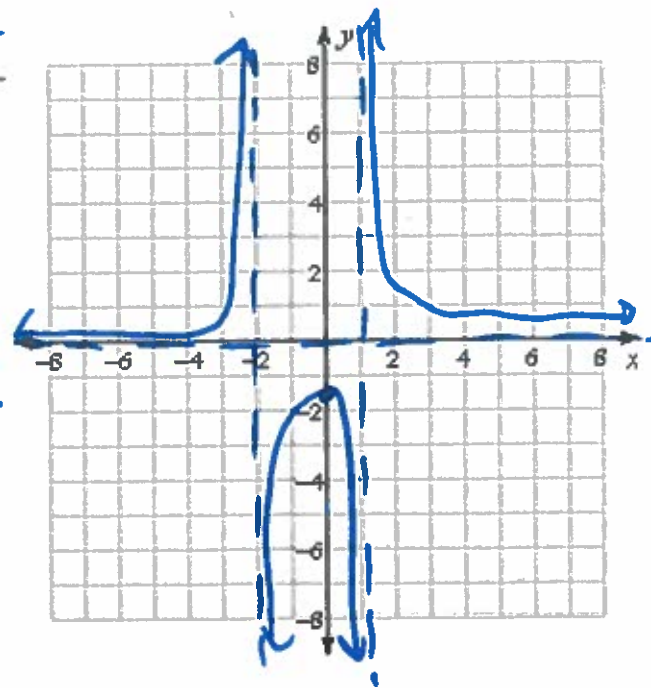
2) Vertical Asymptote(s) $X=-2, X=1$

$$x^2+x-2=0$$

$$(x+2)(x-1)=0$$

3) Holes? None

x	f(x)
-4	3/10
-3	3/4
-1	-3/2
2	3/4



4) End Behavior Asymptote $y=0$ $n=0$ $m=2$ $n < m$

Example 4: Graph the function and identify all key features of the graph.

$$f(x) = \frac{x^2+5x+4}{-2x^2-6x} = \frac{(x+4)(x+1)}{-2x(x+3)}$$

1) x-intercept(s)? $X = -4, X = -1$ y-intercept? None
 $0 = \frac{x^2+5x+4}{-2x^2-6x}$ $y = \frac{0+0+4}{0}$

$$(x+4)(x+1) = 0$$

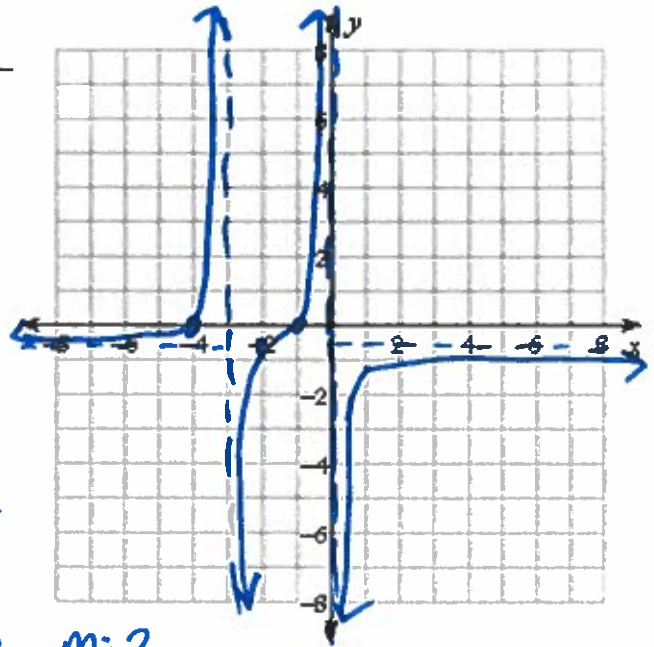
2) Vertical Asymptote(s) $0, -3$

$$\begin{aligned} -2x^2-6x &= 0 & -2x &= 0 & x+3 &= 0 \\ -2x(x+3) &= 0 & x &= 0 & x &= -3 \end{aligned}$$

3) Holes? None

x	f(x)
-5	-1/5
-4	0
-2	-0.5
-1	0
1	-1.25
2	-0.9

$n = 2$ $m = 2$
 $n = m$



4) End Behavior Asymptote $y = \frac{-1}{2}$
only ends are affected

Example 5: Graph the function and identify all key features of the graph.

$$f(x) = \frac{x^3+x^2-12x}{3x^2+3x} = \frac{x(x^2+x-12)}{3x(x+1)} = \frac{x(x+4)(x-3)}{3x(x+1)} \rightarrow X \text{'s simplify}$$

1) x-intercept(s)? $X = -4, X = 3$ y-intercept? None
 $0 = x^3+x^2-12x$ $y = \frac{0}{0}$
 $0 = x(x^2+x-12)$ $x = -4$
 $(x)(x+4)(x-3)$ $x = 3$
 $3x(x+1)$

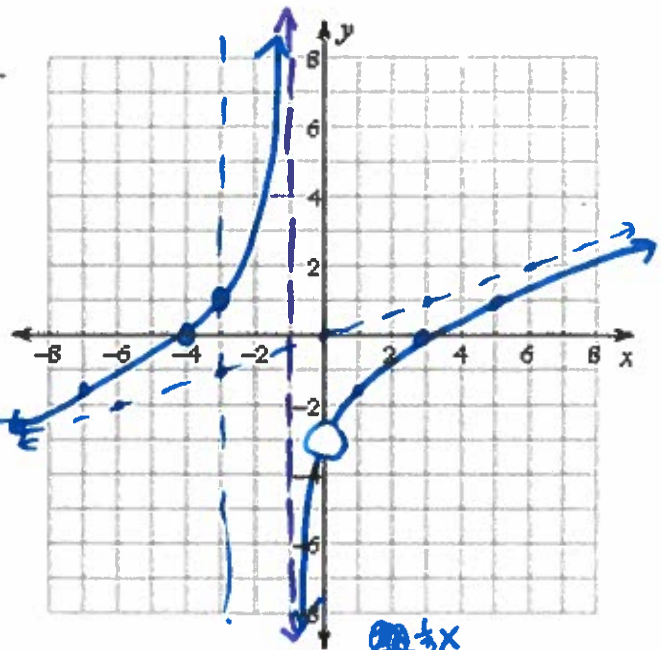
2) Vertical Asymptote(s) ~~$x=0$~~ $x = -1$

$$\begin{aligned} 3x^2+3x &= 0 & x &= -1 \\ 3x(x+1) &= 0 & x &= 0 \end{aligned}$$

3) Holes? $x = 0$

x	f(x)
-9	-2.5
-7	-1.6
-3	1
1	-1.6
5	1

$n = 3$ $m = 2$
 $n > m$



$$\frac{x^3+x^2-12x}{3x^2+3x} = \frac{x^3+x^2-12x}{3x(x+1)}$$

4) End Behavior Asymptote $y = \frac{1}{3x}$

