

Additional Properties of Logarithms

Condense

Product Rule

$$\log_b(MN) = \log_b M + \log_b N$$

Expand

Example 1: Expand

a. $\log_4 12x = \log_4 12 + \log_4 x$

Quotient Rule

$$\log_b(M/N) = \log_b M - \log_b N$$

Power Rule

$$\log_b M^p = p \log_b M$$

b. $\log_7 \frac{19}{x} = \log_7 19 - \log_7 x$

c. $\log_5 (4x)^3$

$$\log_5 (4^3 x^3)$$

$$3 \log_5 4 + 3 \log_5 x$$

d. $\ln \sqrt{x}$

$$\ln x^{1/2}$$

$$\frac{1}{2} \ln x$$

d) $\ln \sqrt{x}$

Example 2: Use the properties of logarithms to expand the following Logarithms:

a. $\log_6 (4x^4y^2)$

$$\log_6 4 + 4\log_6 x + 2\log_6 y$$

b. $\ln \frac{\sqrt[3]{4x+6}}{9}$

$$= \ln \frac{(4x+6)^{1/3}}{9} = \frac{1}{3}\ln(4x+6) - \ln 9$$

c. $\log_b (5x^7\sqrt{y})$

$$= \log_b (5x^7y^{1/2})$$

$$\log_b 5 + 7\log_b x + \frac{1}{2}\log_b y$$

Example 3: Use the properties of logarithms to condense the following logarithms
(Write as a single logarithm).

a. $\frac{1}{2} \log x + 7 \log (3x - 2)$
 $\log x^{1/2} + \log (3x - 2)^7$
 $\log \sqrt{x} (3x - 2)^7$

b. $4 \ln (x - 4) - (\ln 2 + \ln x)$
 $\ln (x - 4)^4 - \ln 2x = \ln \frac{(x - 4)^4}{2x}$

c. $\frac{1}{4} [\log_3 x + 3 \log_3 (x - 4)]$
 $\log_3 (x (x - 4)^3)^{1/4}$
 $\log_3 \sqrt[4]{x (x - 4)^3}$

Change of Base Formula for Logarithms

Allows you to change a logarithm from one base to another

$$\log_b M = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}$$

Example 4: Evaluate

a. $\log_6 40$

b. $\log_{13} 17$