Exponential Modeling

Formulae:

Population Growth

$P = P_0(1+r)^t$

↑ original population

Half-life

$A = A_0(0.5)^{t/h}$ ← half-life

Interest compounded *n* times per year

$A = P\left(1+\frac{r}{n}\right)^{nt}$

Interest compounded continuously

$A = Pe^{rt}$

1. The population of Smallville in the year 1890 was 6250 people. Assuming the population increases at a rate of 2.75% per year;
   a. Estimate the populations in both 1915 and 1940.
   b. Predict the year when the population will reach (or reached) 50,000.

a) $P = 6250(1+.0275)^{25}$ $P = 6250(1+.0275)^{50}$

$P = 12{,}314.$ $P = 24{,}264$

b) $50{,}000 = 6250(1+.0275)^t$

$8 = (1.0275)^t$

$\log_{1.0275} 8 = t$

$76.6 \text{ years} = t$

2. The half-life of a certain radioactive substance is 65 days. There are 3.5g present initially.
   a. Calculate the amount left after 34 days.
   b. Estimate when there will be less than 1g remaining.

a) $A = 3.5(0.5)^{34/65}$

$A = 2.44g$

b) $1 = 3.5(0.5)^{t/65}$

$0.2857 = (0.5)^{t/65}$

$\log_{0.5} 0.2857 = t/65$

$1.807 = t/65$

$\boxed{117.5 \text{ days} = t}$

3. The amount C in grams of carbon-14 present in a certain substance after t years is given by $\boxed{C = 20e^{-ht}}$.
   If $t = 0.0001216$, Estimate the half-life (h) of carbon-14.

$10 = 20e^{-h(0.0001216)}$

$\frac{1}{2} = e^{-h(0.0001216)}$

$\ln\left(\frac{1}{2}\right) = \ln e^{-h(0.0001216)}$

$-0.6931 = -h(0.0001216)$

$-5700.22 = -h$

about 5700 years $= h$

4. If John invests \$2300 in a savings account with a 9% interest rate compounded quarterly, how long will it take until John's account has a balance of \$4150?

$$4150 = 2300\left(1+\frac{.09}{4}\right)^{4t}$$

$$\frac{83}{46} = 1.0225^{4t}$$

$$\log_{1.0225}\left(\frac{83}{46}\right) = 4t$$

$$26.5 = 4t$$

$$\boxed{6.6\text{ years} = t}$$

5. Determine how much time is required for an investment to triple in value if interest is earned at the rate of 6.25% compounded monthly.

$$3000 = 1000\left(1+\frac{.0625}{12}\right)^{12t}$$

$$3 = 1.00521^{12t}$$

$$\log_{1.00521} 3 = 12t$$

$$211.5 = 12t$$

$$\boxed{17.6\text{ years} = t}$$

6. The president of a bank has \$18 million in his bank's investment portfolio that he wants to grow to \$25 million in 8 years.
   a. What interest rate compounded semi-annually does he need for this investment?
   b. What interest rate compounded does he need if interest is compounded continuously?
   c. How long would it take for the investment to double at the interest rate you found in part (a)?
   d. How long would it take for the investment to double at the interest rate you found in part (b)?

a) $25 = 18\left(1+\frac{r}{2}\right)^{2(8)}$

$$\frac{25}{18} = \left(1+\frac{r}{2}\right)^{16}$$

$$\sqrt[16]{\frac{25}{18}} = 1+\frac{r}{2}$$

$$1.0207 = 1+\frac{r}{2}$$

$$.0207 = \frac{r}{2}$$

$$0.0415 = r$$

$$\boxed{4.15\% = r}$$

b) $25 = 18e^{r(8)}$

$$\frac{25}{18} = e^{8r}$$

$$\ln\left(\frac{25}{18}\right) = 8r$$

$$0.0411 = r$$

$$\boxed{4.11\% = r}$$

c) $36 = 18\left(1+\frac{.0415}{2}\right)^{2t}$

$$2 = 1.02075^{2t}$$

$$\log_{1.02075} 2 = 2t$$

$$33.75 = 2t$$

$$\boxed{16.9\text{ years} = t}$$

d) $36 = 18e^{.0411t}$

$$2 = e^{.0411t}$$

$$\ln 2 = .0411t$$

$$\boxed{16.9\text{ years} = t}$$