Composition of Functions Name: _____ Date: _____ Period: ____

F... $f \circ g$ and $g \circ f$, their domains, and evaluate the composition for given value.

1. $f(x) = 2x - 3;\ g(x) = x + 1\ ;\ (f \circ g)(3) and (g \circ f)(-2).$ D: ℝ D: ℝ

$f \circ g = 2(x+1) - 3 = 2x + 2 - 3 = 2x - 1$ (Domain: ℝ) $(f \circ g)(3) = g(3) = 3 + 1 = 4$

$f(4) = 2(4) - 3 = \boxed{5}$

$g \circ f = (2x - 3) + 1 = 2x - 2$ (Domain ℝ)

$(g \circ f)(-2) = f(-2) = 2(-2) - 3 = -7$

$g(-7) = -7 + 1 = \boxed{-6}$

2. $f(x) = x^2 + 4;\ g(x) = \sqrt{x+1}\ ;\ (f \circ g)(-3) and (g \circ f)(2).$ D: ℝ D: $[-1, \infty)$ ← $x + 1 \geq 0$, $x \geq -1$

$f \circ g = (\sqrt{x+1})^2 + 4 = x + 1 + 4 = x + 5$ (Dom: $[-1, \infty)$) $(f \circ g)(-3) \Rightarrow$ Not possible (-3 is not in the domain)

$g \circ f = \sqrt{(x^2+4)+1} = \sqrt{x^2+5}$ (Dom: $[-1, \infty)$)

$(g \circ f)(2) = f(2) = 2^2 + 4 = 8$

$g(8) = \sqrt{8+1} = \sqrt{9} = \boxed{3}$

3. $f(x) = x^2 - 1;\ g(x) = \frac{1}{x-1}\ ;\ (f \circ g)(-1) and (g \circ f)(4).$ D: ℝ D: $(-\infty, 1) \cup (1, \infty)$

$f \circ g = \left(\frac{1}{x-1}\right)^2 - 1 = \frac{1}{(x-1)^2} - 1$ (Dom: ℝ, $x \neq 1$) $(f \circ g)(-1) = g(-1) = \frac{1}{-1-1} = -\frac{1}{2}$

$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 - 1 = \frac{1}{4} - 1 = \boxed{-\frac{3}{4}}$

$g \circ f = \frac{1}{(x^2-1)-1} = \frac{1}{x^2-2}$ (Dom: ℝ, $x \neq \pm 1$ or $\pm\sqrt{2}$) $(g \circ f)(4) = f(4) = 4^2 - 1 = 15$

$g(15) = \frac{1}{15-1} = \boxed{\frac{1}{14}}$

4. $f(x) = \frac{1}{2x};\ g(x) = \frac{1}{3x}\ ;\ (f \circ g)(0) and (g \circ f)\left(-\frac{2}{3}\right).$ D: $x \neq 0$ D: $x \neq 0$

$f \circ g = \frac{1}{2\left(\frac{1}{3x}\right)} = \frac{1}{\frac{2}{3x}} = \frac{3x}{2}$ (Dom: ℝ, $x \neq 0$) $(f \circ g)(0) \Rightarrow$ Not possible, $x \neq 0$

$g \circ f = \frac{1}{3\left(\frac{1}{2x}\right)} = \frac{1}{\frac{3}{2x}} = \frac{2x}{3}$ (Dom: ℝ, $x \neq 0$) $(g \circ f)\left(-\frac{2}{3}\right) = f\left(-\frac{2}{3}\right) = \frac{1}{2\left(-\frac{2}{3}\right)} = \frac{1}{-\frac{4}{3}} = -\frac{3}{4}$

$g\left(-\frac{3}{4}\right) = \frac{1}{3\left(-\frac{3}{4}\right)} = \frac{1}{-\frac{9}{4}} = \boxed{-\frac{4}{9}}$

5. $f(x) = x^3;\ g(x) = \sqrt[3]{1 - x^3}\ ;\ (f \circ g)(0) and (g \circ f)(1).$ D: ℝ D: ℝ

$f \circ g = \left(\sqrt[3]{1-x^3}\right)^3 = 1 - x^3$ (Dom: ℝ) $(f \circ g)(0) = 1 - 0^3 = \boxed{1}$

$g \circ f = \sqrt[3]{1-(x^3)^3} = \sqrt[3]{1-x^9}$ (Dom: ℝ) $(g \circ f)(1) = \sqrt[3]{1-1^9} = \sqrt[3]{0} = \boxed{0}$

Application Problems

6. A commercial bakery makes a mango meltaway cookie. The cost to make the cookie depends on the diameter of the mango pit. The size of mango pit depends the average temperature during the growing season.

The cost to manufacture the cookie is ***c = f(m)***

$f(40) = .012$

m Size of Mango Pit in millimeters	20	30	40	50	60	70
c Cookie Cost per cookie in dollars	.005	.008	.012	0.17	.023	.030

The size of the pit is a function of temperature ***m= g(t).***

t Temperature in °F	80°	85°	90°	95°	100°	105°
m Size of Mango Pit in millimeters	18	24	32	40	50	62

$g(95) = 40$

Use the tables to evaluate $f(g(95°))$ $= .012$

7. The size of a baby tarantula depends upon the number of eggs laid by the mother. The number of eggs laid by the mother depends upon the age of the mother. ***s = f(n)***

n number of eggs	100	120	140	160	180	200
s size of tarantula (in.)	10	9	8	7	.6	5

$f(120) = 9$

The size of the pit is a function of temperature ***n= g(a).***

a Age in years	4	8	12	16	20	24
n number of eggs	100	120	140	160	180	200

$g(8) = 120$

$f(g(8)) = 9$

Use the tables to evaluate *f(g(8)*

8. The number of cherry tomatoes produced by a single plant depends upon the amount of acid found in the soil. The amount of acid in the soil depends upon the zone that soils is located in. ***t = f(a)***

a % acid	10	15	20	25	30	35
t number of tomatoes	240	200	160	120	80	40

$f(15) = 200$

The size of the pit is a function of temperature ***a= g(z).***

z zone	1	2	3	4	5	6
a acid	35	30	25	20	15	10

$g(5) = 15$

Use the tables to evaluate *f(g(5)* $= 200$

9. Find *f*(*x*) and g(*x*) so that *h*(*x*) = *f*(*g*(*x*)). (There may be more than one answer.) DECOMPOSE

a. $h(x) = \sqrt{x-7}$ b. $h(x) = (x^3-3)^2$ c. $h(x) = \frac{2}{\sqrt{3x+5}}$ d. $h(x) = \left|(\sqrt{x}-3)^2\right|$

g(x) = $x-7$ g(x) = x^3-3 g(x) = $3x+5$ g(x) = $(\sqrt{x}-3)^2$

f(x) = $\sqrt{x}$ f(x) = x^2 f(x) = $\frac{2}{\sqrt{x}}$ f(x) = $|x|$

10. Find *g*(*f*(*x*)) and *f*(*g*(*x*)) for $f(x) = \frac{3}{4x}$ *and* $g(x) = \frac{x}{2x}$.

$g(f(x)) = \frac{1\left(\frac{3}{4x}\right)}{2\left(\frac{3}{4x}\right)} = \frac{1}{2}$

$F(g(x)) = \frac{3}{4\left(\frac{x}{2x}\right)} = \frac{3}{2}$