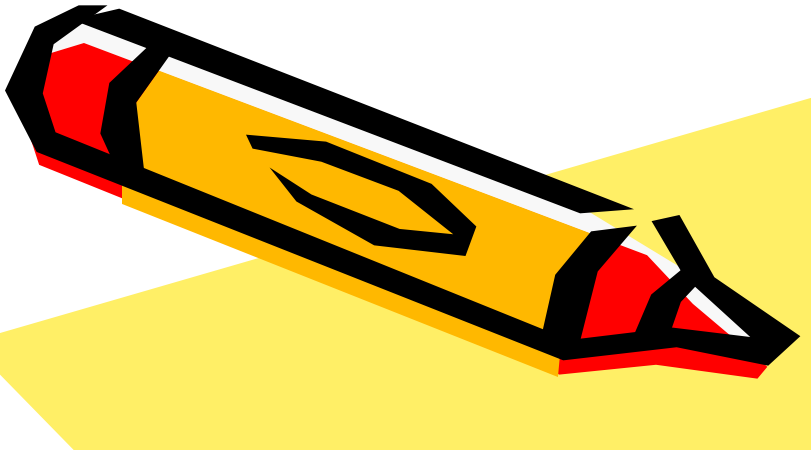

Honors PreCalculus

Sequences

What's a sequence?

- An ordered progression of numbers
  - Finite sequence: 2,4,6,8,10
  - Infinite sequence: $1, 3, 9, \ldots, 3^k, \ldots$
  - Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, . . .

What's a sequence?

- Arithmetic sequence
  - Numbers in the sequence have a common difference
    - Add or subtract the same number each time
  - General "rule"
    - $a_n = a_1 + (n - 1)d$

a_1 → first number

d → common difference

Arithmetic Sequences

Example 1:

Find the common difference, the rule (equation) and the 10th term of the sequence: -3, -1, 1, 3, 5, . . .

- Equation (rule): $a_n = a_1 + (n - 1)d$ +2 +2 +2
  - $a_1 = -3$
  - $d = 2$
  - So: $a_n = -3 + (n - 1)(2) = -3 + 2n - 2 = 2n - 5$
  - 10th term: $n = 10$, so $a_{10} = 2(10) - 5 = 20 - 5 = 15$

20-5

15

Arithmetic Sequences

Example 2:

Find the common difference, the rule (equation) and the 10th term of the sequence: 6, 2, -2, -6, -10, . . .

- Equation (rule): $a_n = a_1 + (n - 1)d$
  - $a_1 = 6$
  - $d = -4$
  - So: $a_n = 6 + (n - 1)(-4) = 6 - 4n + 4 = -4n + 10$
  - 10th term: $n = 10$, so $a_{10} = -4(10) + 10 = -40 + 10 = -30$

Constructing Sequences

Example 3: $a_n = a_1 + (n-1)d$

If the 2nd and 5th terms of a sequence are 3 and 24, respectively, find the equation of the arithmetic sequence

$a_2 = 3 \longrightarrow 3 = a_1 + (2-1)d \rightarrow 3 = a_1 + d \rightarrow 3 - d = a_1$

$a_5 = 24 \longrightarrow 24 = a_1 + (5-1)d$

$24 = 3 - d + 4d$

$24 = 3 + 3d$

$21 = 3d$

$7 = d$

$3 - 7 = a_1$

$-4 = a_1$

$a_n = -4 + (n-1)7$

$a_n = 7n - 11$

What's a sequence?

- Geometric sequence
  - Numbers in the sequence have a common ratio
    - Multiply or divide by the same number each time
  - General "rule"
    - $a_n = a_1(r^{n-1})$

Geometric Sequences

Example 4:

Find the common ratio, the rule (equation) and the 10^{th} term of the sequence: 3, 6, 12, 24, 48, . . .

- Equation (rule): $a_n = a_1(r^{n-1})$
  - $a_1 = 3$
  - $r = 2$
  - So: $a_n = 3(2^{n-1})$
  - 10^{th} term: n = 10, so $a_{10} = 3(2^{10-1}) = 3(2^9) = 1536$

Constructing Sequences

Example 5:

If the 2nd and 5th terms of a sequence are 3 and 24, respectively, find the equation of the geometric sequence

Limit of a sequence

A sequence converges if the numbers in the sequence approach a particular number

A sequence diverges if the numbers in the sequence approach infinity, or don't actually approach any particular number

Limit of a sequence

Examples: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{1}{n}, \ldots$ Converges to 0

$\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots \frac{n+1}{n} = 1 + \frac{1}{n}$ Converges to 1

$2,4,6,8,10,\ldots$ Diverges

$-1,1,-1,1,\ldots,(-1)^n,\ldots$ Diverges