

Angle Measure and Trigonometric Functions

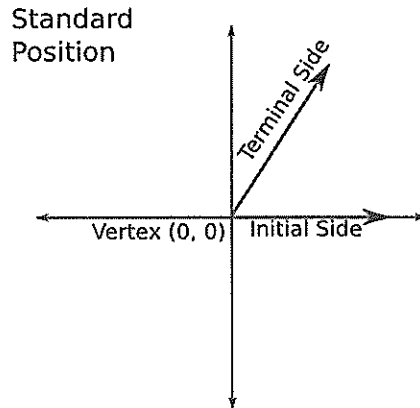
- Students will place angles in standard position and differentiate between positive and negative angle rotations

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Standard position of non-acute angles

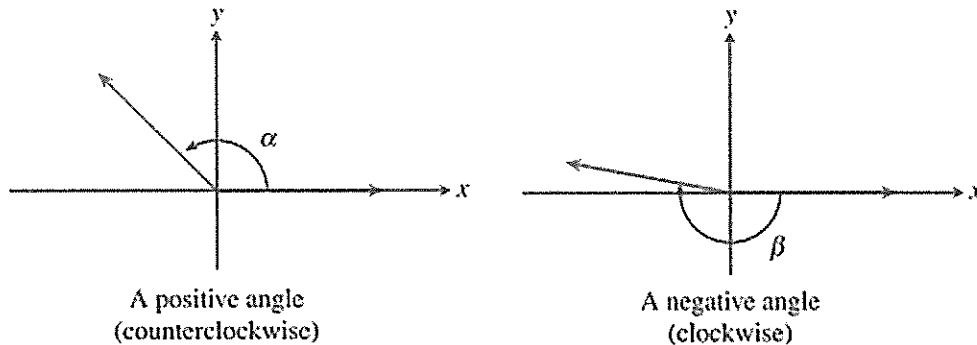
Standard position of an angle on the coordinate plane:

- Initial side on the positive x axis
- Terminal side can be in any quadrant



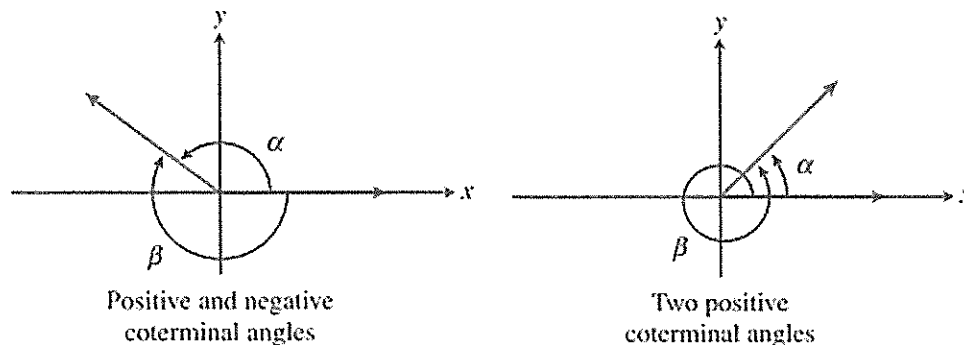
Rotation of Angles

Angle Rotation: Positive vs. Negative



Coterminal angles

Coterminal angles: Angles that have the same initial side and the same terminal side, but different measures.



Angle Measure and Trigonometric Functions

- Students will find coterminal angles

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Example 1: Sketch the angle in Standard Position, then find values for (and sketch) one positive and one negative coterminal angle for each of the following:

a. 60°

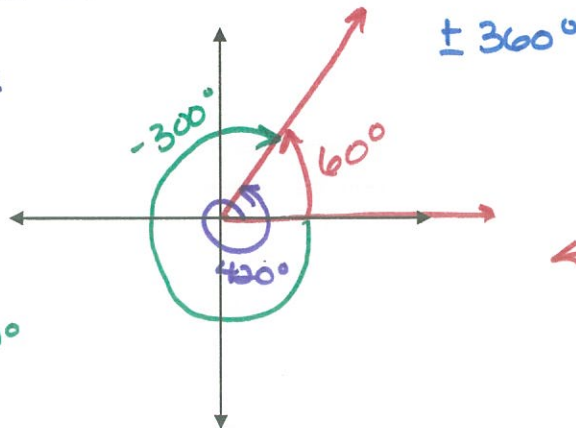
b. -175°

POSITIVE COTERMINAL:

$$60^\circ + 360^\circ = 420^\circ$$

NEGATIVE COTERMINAL:

$$60^\circ - 360^\circ = -300^\circ$$

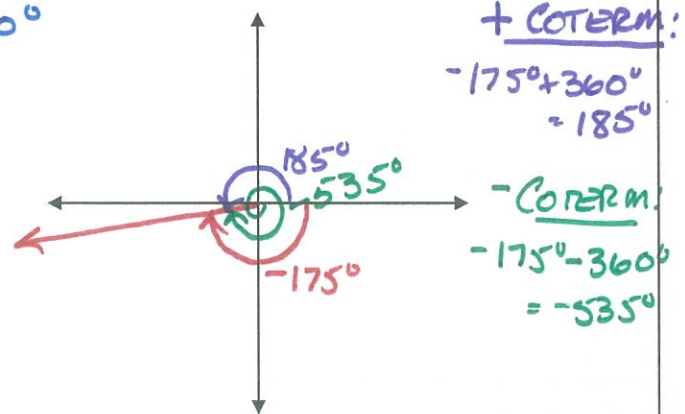


+ COTERM:

$$-175^\circ + 360^\circ = 185^\circ$$

- COTERM:

$$-175^\circ - 360^\circ = -535^\circ$$



c. $\frac{4\pi}{7}$

$\pm 2\pi$

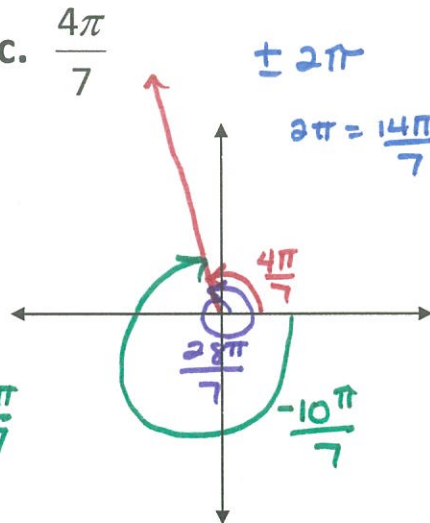
$$2\pi = \frac{14\pi}{7}$$

+ COTERM:

$$\frac{4\pi}{7} + \frac{14\pi}{7} = \frac{28\pi}{7}$$

- COTERM:

$$\frac{4\pi}{7} - \frac{14\pi}{7} = -\frac{10\pi}{7}$$



d. $-\frac{2\pi}{5}$

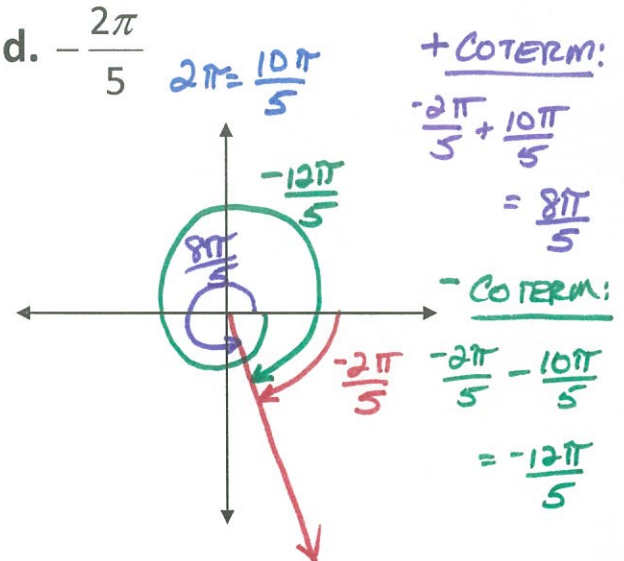
$$2\pi = \frac{10\pi}{5}$$

+ COTERM:

$$-\frac{2\pi}{5} + \frac{10\pi}{5} = \frac{8\pi}{5}$$

- COTERM:

$$-\frac{2\pi}{5} - \frac{10\pi}{5} = -\frac{12\pi}{5}$$



Just a reminder:

Complimentary angles

Sum up to 90° or $\frac{\pi}{2}$

Quadrantal Angles:

Angles whose terminal side lies along an axis.

Supplementary angles

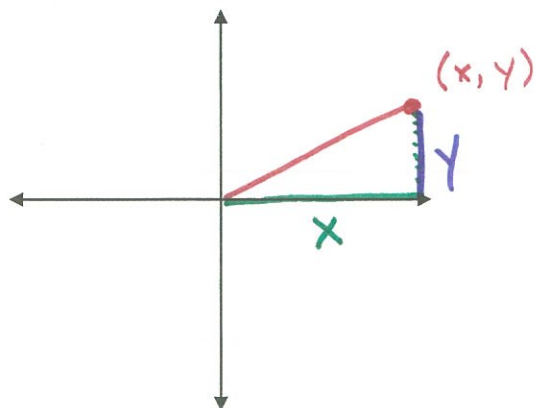
Sum up to 180° or π

Angle Measure and Trigonometric Functions

- Students will find values for all six Trigonometric Functions for angles in any quadrant on a coordinate system

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Values and signs of the Trigonometric functions in any quadrant:



$$x^2 + y^2 = r^2$$

so:

$$r = \sqrt{x^2 + y^2}$$

What are reciprocal functions?

$$(x, y) = (\cos, \sin)$$

"Standard" Trigonometric Functions

Sine $\sin \theta = \frac{y}{r}$

Cosine $\cos \theta = \frac{x}{r}$

Tangent $\tan \theta = \frac{y}{x}$

Reciprocal Trigonometric Functions

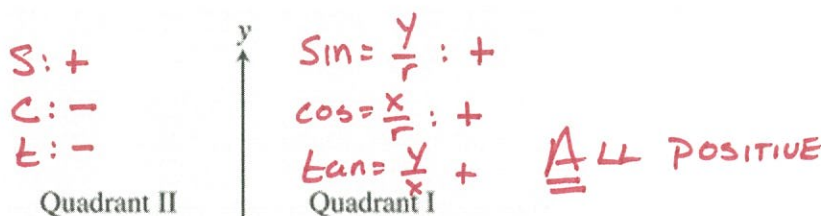
Cosecant $\csc \theta = \frac{r}{y}$
 $\frac{1}{\sin}$

Secant $\sec \theta = \frac{r}{x}$
 $\frac{1}{\cos}$

Cotangent $\cot \theta = \frac{x}{y}$
 $\frac{1}{\tan}$

Where are trig functions positive?

SIN +



TAN +

S: -
 C: -
 T: +
 S: -
 C: +
 T: -
Cos +

- Students will find values for all six Trigonometric Functions for angles in any quadrant on a coordinate system

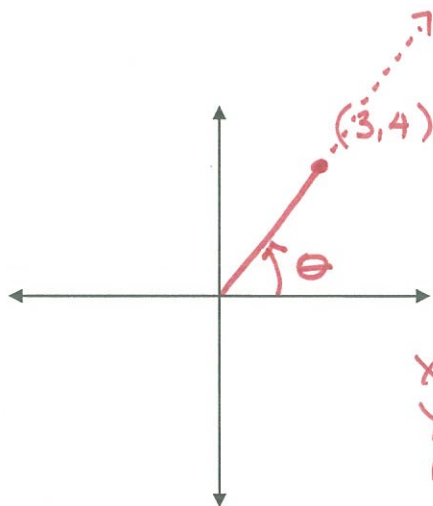
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Finding values of Trig functions given an ordered pair

Example 2: For the angles whose terminal sides pass through the given point, determine in which quadrant the terminal side of the angle lies, sketch the angle in standard position, and evaluate all six Trigonometric functions of that angle.

a. (3, 4)

$$r = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$



$$\begin{aligned} x &= 3 \\ y &= 4 \\ r &= 5 \end{aligned}$$

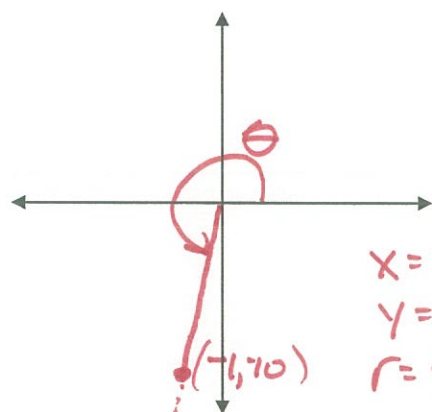
$$\sin \theta = \frac{4}{5} \quad \csc \theta = \frac{5}{4}$$

$$\cos \theta = \frac{3}{5} \quad \sec \theta = \frac{5}{3}$$

$$\tan \theta = \frac{4}{3} \quad \cot \theta = \frac{3}{4}$$

b. (-1, -10)

$$r = \sqrt{(-1)^2 + (-10)^2} = \sqrt{101}$$



$$\begin{aligned} x &= -1 \\ y &= -10 \\ r &= \sqrt{101} \end{aligned}$$

$$\sin \theta = \frac{-10}{\sqrt{101}} = -\frac{10\sqrt{101}}{101} \quad \csc \theta = -\frac{\sqrt{101}}{10}$$

$$\cos \theta = \frac{-1}{\sqrt{101}} = -\frac{\sqrt{101}}{101} \quad \sec \theta = -\sqrt{101}$$

$$\tan \theta = \frac{-10}{-1} = 10 \quad \cot \theta = \frac{1}{10}$$

Angle Measure and Trigonometric Functions

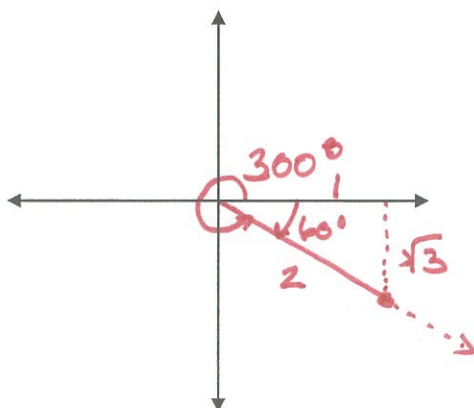
- Students will find values for all six Trigonometric Functions for angles in any quadrant on a coordinate system

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Finding values of Trig functions given an ordered pair

Example 3: Find the values for the six Trig functions for the following angles:

a. 300°



$$x = 1 \quad y = -\sqrt{3} \quad r = 2$$

$$\sin \theta = \frac{-\sqrt{3}}{2}$$

$$\csc \theta = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

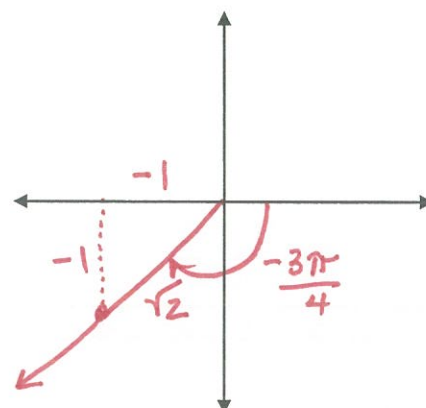
$$\cos \theta = \frac{1}{2}$$

$$\sec \theta = 2$$

$$\tan \theta = -\sqrt{3}$$

$$\cot \theta = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

b. $-\frac{3\pi}{4}$



$$x = -1 \quad y = -1 \quad r = \sqrt{2}$$

$$\sin \theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\csc \theta = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

$$\cos \theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\sec \theta = -\sqrt{2}$$

$$\tan \theta = \frac{-1}{-1} = 1$$

$$\cot \theta = 1$$

Angle Measure and Trigonometric Functions

- Students will find values for one trigonometric function when given the value of a second trigonometric function

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Using one Trig function to find others.

Example 4:

If $\overset{+}{\sin}\theta = \frac{4}{9}$ and $\overset{-}{\tan}\theta < 0$, find $\overset{-}{\cos}\theta$ and $\overset{-}{\tan}\theta$

$$\begin{aligned}y &= 4 & r &= 9 & x^2 + 16 &= 81 \\x^2 & & & & x^2 &= 65 \\x & & & & x &= \sqrt{65}\end{aligned}$$

$$\overset{-}{\cos}\theta = -\frac{\sqrt{65}}{9} \quad \overset{-}{\tan}\theta = -\frac{4}{\sqrt{65}} = -\frac{4\sqrt{65}}{65}$$

Example 5:

If $\overset{+}{\csc}\theta = \frac{7}{3}$ and $\overset{+}{\cos}\theta > 0$, find $\overset{+}{\cos}\theta$ and $\overset{+}{\cot}\theta$

$$\overset{+}{\sin}\theta = \frac{3}{7} : y = 3 \quad r = 7$$

$$x^2 + 9 = 49$$

$$x^2 = 40$$

$$x = \sqrt{40} = 2\sqrt{10}$$

$$\overset{+}{\cos}\theta = \frac{2\sqrt{10}}{7} \quad \overset{+}{\cot}\theta = \frac{x}{y} = \frac{2\sqrt{10}}{3}$$